Rebuttal letter

We thank both reviewers for their work that have helped us to improve the manuscript. We have done our best to answer their questions.
Some confusion came from the fact that we sometimes wrongly used the work "anomalies" instead of "heterogeneities" which definitely didn't help the understanding of our point. We refer to "anomalies" for DAS signals that are not consistent with the wave propagation (typically too small spatially for a given frequency). Heterogeneities are regions with mechanical properties different from a background.
Another significant source of confusion arose from the misconception that the numerical tests were designed to replicate the data presented in Section 3. However, this is not the case. The paper's methodology involved initially developing the numerical model, followed by seeking an actual data example to illustrate our findings. We have made efforts to clarify this aspect to eliminate any potential confusion.

1 Reviewer 1

- 1 17-18 might be simplified
  we have tried to improve this part
- 2 31-32 hard to read
  we have tried to improve this part.
- 3 37 not a density
  linear density?
- 4 44 not clear which ones !
  we mean : 1: simply pulled in underground conduits 2: the FO is decoupled from the outer jacket
- 5 45 not clear
  We have tried to improve that
- 6 61-62 here there is something unclear. It has never be said before that DAS is a measurement over an extended distance, then the opposition of strain and point measurement is not straightforward
  Our mistake: a few words were missing which makes the sentence have no meaning. We added "... particle displacement or velocity point measurements." Sorry about that.
- 7 74 not sure of that, do you have a reference ? Their are also Brioullin and Raman non elastic scattering. but Rayleigh scattering is what is analyzed

  We removed the word "dominant" which is not critical for the discussion
- 8 79 strain (Dl/l), where elongation is Dl
  done
- 9 83 difficult to read
  changed
- 10 86 exponent -12 lacking Hz should not be italic I think the distance is also required
  Done
- 11 95 receiver spacing and gauge length are independant I think it was fixed in first version of Febus but it is not always the case

  Corrected. The gauge length here was actually 9.6 m

- 12 102 add ref for this formula. also is xi+1 ¿xi the dt should be negative: the optical wave reflect on xi+1 and interfere at xi with a delay, then the optical delay is on ux and dt should be explicitely given in function of the distance difference and c. this formula need to be explained with more detailled as it is the basis of the paper

  We changed the formula and added a reference

- 13 106 the rayleigh scattering is not evaluated in DAS
  done

- 14 113 epicenter position?
  Done

- 15 124 mixed between singular and plural please correct
  corrected

- 16 126 is that the black lines ? it is perturbating to refer to 900 sensors better add between the distances 8000? to 10000 m because no reference of the sensor number is given in fig1
  Indeed, it is the black lines. We have changed the text to refer to the black lines.

- 17 127 very difficult to understand color of the filter seems to be inverted (red signals contains higher frequencies (even if seen in space)) Also the waveforms seems to be normalized in the representation, then you can refer to attenuation.
  We admit it is not obvious, but the color scale is not inverted. The main reason for this confusing impression is the anomalies that are dominating the signal. We have added a magenta box in fig1(c) to highlight one example of signal we wish to show.

- 18 129-131 not clear
  We have tried to improve this part

- 19 144 Is that related to the method you use to solve the elasto dynamic problem? I do not think it is written where it should be.
  Maybe we are missing the point of this comment. When defining the problem, we need to specify the boundary conditions used. Here, we neither use Dirichlet nor Neumann boundary conditions, but absorbing boundaries. Those are meant to absorb any outgoing waves. We are not stating yet what method we are using to implement those boundary conditions.

- 20 151 does look like rather a monopole /explosion than a usual vector point source
  This is a standard expression for a moment tensor point source. See, for example, Dahlen & Tromp (1998) Eq. 5.85 page 166. Please note that the gradient operator applies to the Dirac distribution.

- 21 155 indeed see comment at line 144-145
  See above answer

- 22 156 constante?
  yes

- 23 160 please check compatibility with eq 2
  done

- 24 160 mainly dt disappeared in first term
  It did because we are using v instead of u. But, this is a good point, we change Eq. 2 to be the same as Eq 6.

- 25 161 not in the equation 6
  Not sure about the point of this comment. Δ_x is in Eq. 6.

- 26 172 vp is too low, see brocher relation to fix that
Indeed, we used a V_P that we can find in dry sand or topsoil (The first author had in mind a FO in volcano hashes at the very beginning, and it unfortunately stayed that way). Because changing that would imply re-running everything, re-do all the figures without changing the visual aspect of the figures nor any conclusions of this work, we didn't change that value. We removed the sentence "representative of subsurface layers" as it is not accurate.

- 27 175 I m lost here. You never mentionned 2 fibers before. Also fibers are on the surface not in the bulk as in your simulation. Then shoul I understand that you simulated only surface waves or disregard the effect of surface wave reflection on your analysis?
Indeed, the numerical experiment presented here should not be seen as an attempt to model the sec. 3 real data. Nevertheless, such a simple modeling is sufficient to reach our point: the effect of small scale heterogeneities on DAS measurement. And yes, we are using two FO cables in our numerical experiment to make our point.

- 28 178 then it is not to mimic the FO cables?
See the above answer: it is to illustrate FO cables in general, not specifically the one of the previous observations in Sec. 3.

- 29 fig 3 can we see the effect on S wave too ?
Because the chosen source is an explosion, there is no ballistic S-wave. There are S waves in the scattered wavefield after the ballistic P-wave, but they are difficult to isolate.

- 30 188 Why rock are discretized by only 2 cells? Don t you require at least 6 cells for an heterogeneity to ensure that the errors is not numerical? The diffracted field by rock is it well sampled close the rock? Perhaps in farefield but certainly not in nearfield. I think the convergence close to the diffractors should be shown in supplementary at least. You also need to discuss the error in displacement and in strain, I think they differ from 1 order due to the derivative. What is the ratio impose between element length and lambda_min?
A general fact of the dynamic is that, to obtain good quality result, it is enough to mesh the geometry (discontinuities) and the minimum wavelength. This is not true for the static case, for which singularities of the solution near complex geometry implies a finer and finer mesh. Knowing that, a single element per stone would be enough. We have 4 elements/stone here because the meshing tool (GMSH) we use here is not smart enough to join the 4 elements into 1. Another meshing tool (CUBIT and its successor, commercial) would have been able to generate a mesh with a single element par stone. The polynomial degree used here is 6, which means each element has 7x7 "points" (Gauss-Lebatto-Legendre collocated points). Such a degree is enough to accurately model any wavefield using 1 element per minimum wavelength. Here, because we have to mesh the stones, the elements are very small compared to the minimum wavelength (in a smooth media, the elements could be 15 times larger, the wavefield would still be accurate). The wavefield is therefore seriously over-sampled, which, together with the well-known accuracy of the SEM, ensures we have no accuracy issue here. If you are interested in knowing more about the spectral element convergence, there are many studies, but one of the first is Seriani & Priolo (1994).

- 31 190-194 Why not introducing g dot in 5 and use v and strain rate? The explanation even if right is complicating the paper.
We have thought about that and what you suggest was indeed a considered option. But this discussion about g depends on the nature of the source (g is a step function for an earthquake, a Dirac for a triggered explosion etc). Because in the end it doesn't change much the results and because its application is not tied to the source nature, we decided to keep it that way.

- 32 197 one case 2 variables this is confusing because there are 2 fibres,
We changed that

- 33 197 we only see from 26 s, change the description of first
  The signal is zero before 26s, this is the first arrival. It took about 25s for the P waves to reach the FO1 (+2s to account for the g zero time).

- 34 202 this is only from here I can understand what anomalies are in section 3 However I am not very convinced. In fig3 du, the balistic wave has a dicontinuous amplitude over the different chanel and it iseems more related to channel normalization. The amplification might be related to local site effect, and here you are in a bulk.
  We are not sure what is meant here. About the normalization: it is done by maps, not by traces, so that the relative amplitudes within the maps are preserved. Moreover, the cross-section for a given time step in Fig 5 (formerly fig 4) right panel shows clear "glitches" (anomalies) and only for the DAS "strain" measurement, not for the displacement (fig 5 left panel). This observation can indeed be called a "site effect", but then it is an effect only for a strain measurement, not for a displacement measurement at the same location. Classical "site effects" are ringing effects that still obey a dispersion relation (they appear only in some frequency bands), which is not the case here.

- 35 Fig 6 a portion of
  The caption has been updated

- 36 Fig 7 It is not clear that the effect is lower on FO2 as everyting is normalized. Can you show it also not normalized?
  The traces are now normalized by a common factor (the maximum amplitude of FO1) which makes the amplitude comparable.

- 37 267 I am not sure about the shortcut.
  "Similarly" has been removed

- 38 271 better use index at left ?
  This is possible. Nevertheless, this notation used here is very common for many two-scale homogenization works.

- 39 310 Eq?
  Done

- 40 Fig 8 Is it a 1D material ? 1D homogeneisation?
  Fig 9 (previously Fig 8) is a cross-section of the 2-D homogenized solution. The method employed is the one described in Section 5. See the beginning of section 5.3.

- 41 380 The effect of filtering is not given explicitly in section 3. Is that for changing wavelength?
  Exactly. Following the dispersion relation, changing the frequency should change the wavelength. If it is not the case, the observed signal is an anomaly. We have tried to improve section 3 accordingly.

- 42 381 Not clear, DAs measurement shows anomalies. There are not affected by. Or do you mean the diffractors? Did you verify where the vertical lines stand that there are shallow heterogeneities?
  This is our mistake. We meant "display anomalies". This has been corrected and the paragraph rewritten.

- 43 402 I disagree, see site effects due to free surface topography.
  We are not sure why it is a disagreement. Indeed, free surface fine-scale topography has a similar effect compared to fine-scale heterogeneities regarding displacement versus strain. See Capdeville & Marigo (2013).

2 Reviewer 2

- 1. Page 2, Line 39 : What do the authors mean by cable coating or coupling? Kindly elaborate.
  We replaced "coating" by "cable jacket" and rephrased the sentence as: "The attachment of the Fiber Optic (FO) cable to both its cable jacket and the ground is known to strongly affect DAS measurements "

- 2. Page 3, Line 80 : What does the parameter ξ signify in Equation 1? Explained after the equation: " ξ is assumed constant, typically 0.79 in single-mode fiber, and acknowledges that straining the fiber also implies a proportional change in its refractive index". The reader can refer to the reference provided for the equation to get more details.

- 3. Page 3, Line 88 : Kindly elaborate on what the authors mean to convey in the statement "But some experiments on short fiber segments exploited data acquired at several kHz". This sentence is indeed not critical and was removed.

- 4. Page 4, Line 111 : Please provide further details (Date, Time, Coordinates) of the 2.7Mw aftershock recorded by the DAS fiber optic cable Done

- 5. Page 5 : Please specify the lengths of the two arms of the V-shaped network in Figure 1. The data shown in Figure 1 correspond to which time of the year approximately?

  Done. Figure 1 corresponds to the date the aftershock studied i.e. Nov. 23rd, 2019

- 6. Page 4, Line 123 : Is there a specific reason for choosing the 4 high-frequency cutoff? Why was it not just 5, 10, 15, 20 and 25?

  We tried different values and found that the chosen high-frequency cutoff frequencies to better illustrate the attenuation of the seismic waves with respect to the anomalies.

- 7. Page 4, Line 130-131 : The authors state that they do not observe the effect of filtering over one or few channels. Is it possible for the authors to mark at least one such region in Figure 1c to back this statement?
  We added a white box in Fig 1(b) and a cyan box in Fig 1(c) to display one example.

- 8. Page 4, Line 132: What do the authors mean by 'environment of the fiber'. Kindly elaborate.
  We mean anything near the fiber. We elaborated on the sentence

- 9. Page 4, Line 133 : When do the two anomalies, phase unwrapping errors and laser frequency drifts, occur in DAS acquisition?

  We provided examples: "Two well-known sources of anomalies inherent to DAS technology are phase unwrapping errors (e.g. something directly touching the OF and causing a sudden large strain) and laser frequency drifts (e.g. if the laser of the DAS is affected by external vibrations)"

- 10. Page 6, Line 139 : What do the authors mean by mechanical property anomalies? Please elaborate on this term before the paper delves into the numerical analysis.
  Sorry, we meant "heterogeneities" instead of "anomalies". This error is embarrassing as it is very confusing. Indeed, we observe anomalies in DAS measurements that we propose to link to small-scale heterogeneities in our work.

- 11. Page 6, Line 159 : What is pulse width effect? Kindly elaborate.
  The pulse width effect is explained just before section 3 and is expected to be small. We have moved the sentence "We ignore the pulse width effect" there for clarity.

- 12. Page 6, Line 160: In Equation 6, does 'z' represent depth of the medium? If not, the reviewer requests the authors to replace 'z' with some other variable.
  The 'z' component represents more a horizontal component than a depth. Nevertheless, the 2-D modeling we are using here only represents partly a 3-D realistic setting and the $(\hat{x}, \hat{z})$ are just the basis vector of this 2-D setting, which is arbitrary and a common practice for numerical modeling. Moreover, we have used z instead of y to avoid confusion with the homogenization $\mathbf{y}$ variable. We have added a sentence to make sure there is no confusion.

- 13. Page 7 : Figure 2 is extremely confusing to the readers. As the y-axis in 'Figure 2a' is marked 'z', it is presumed that it represents the depth of the region of simulation. If so, then the origin of the region should be at top left corner instead of bottom left corner and the depth should be represented as -10km, -20km and so on. In that case it is easy for the reader to perceive that the explosion occurs at a depth of 30km from the top surface and the measurements are being recorded at a distance of 10km approximately. However, DAS fibre at a depth of 10km does not make sense. If 'z' is not the depth of the medium, then how is the simulation carried out? Is the source also assumed to be present at the same level as the DAS? The authors need to clarify on these points.
  We have added a text at the beginning of section 2 to clarify this point. As mentioned earlier, we present only a 2-D model in an infinite plate that can not represent many aspects of the 3-D real setting. We have added a section in the discussion too, about the limitations of the presented modeling.

- 14. Page 7, Line 178 : Either FO1 is directly in contact with 6 stones and FO2 is with none (according to current Figure 2b) or Figure 2b needs to be checked if there are 6 or 7 stones present on FO1.
  Indeed, Fig. 2b represents only a portion of the whole area. This portion is presented with a dotted line square in Fig. 2a. In Fig. 2b, only 6 stones are visible, but a 7th one is touching FO1. We added a sentence in the text to be more precise.

- 15. Page 7, Line 179 : When the authors write, 20km away from the cables, do they mean at a horizontal distance of 20km or a depth of 20km.
  As explained above, it is a 2-D modeling. We mean 20km away in the Ω plane. If we think of this 2-D modeling as a top view of a 3-D modeling, it is indeed 20km in the horizontal distance.

- 16. Page 8, Line 180-181 : Please provide a figurative representation of the Ricker source-time function showing the central and maximum frequency used to simulate the explosive source.
  A figure has been added. (Ricker functions are quite commonly used for numerical modeling in seismology)

- 17. Page 8, Line 186 : Kindly elaborate on what is GMSH. Also, please provide it's full-form.
  GMSH is an open-source software. We added a sentence saying so. The necessary information to know more about it comes with the provided reference.

- 18. Page 8 : Is it possible for the authors to add a subplot of FO1 showing the location of the 7 stones right below both the subplots of Figure 3? It will improve the understanding of the reader to compare the not-so strong and strong glitches on the displacement and strain fields respectively with the actual location of heterogeneities on FO1.
  Done.

- 19. Page 8, Line 201-202 : Please mark the regions in Figure 1c, where authors think that they observe this phenomenon.
  done with a white box in Fig. 1(c)

- 20. Page 8, Line 203 : Kindly elaborate on trace collection representation.
  Added in the figure caption

- 21. Page 9 : It will also be good if the authors can mention the exact distance of these 6/7 stones on the x-axis with respect to the origin. It will help to match these distances with the peaks observed in the blackline in Figure 5.
  The positions of the stones along the FO as been marked by vertical gray lines in Fig 6 (formerly 5) and 8 (formerly 7)/

- 22. The authors are requested to show Figure 2b in the range of 6km to 14km so that this representation is consistent with their observations shown in Figure 3. It is also advisable to mark a small region around FO1 and FO2 which signifies what they show in Figure 4 and Figure 6.
  It is not easy the change the x-range without making the figure difficult to visualize with a standard resolution. Instead, we have added two panels showing the stone locations just below the traces collections.

- 23. The captions for Figure 4 and 6 are incomplete.
  Indeed, thank you. We have completed the caption

- 24. Page 10: It will be interesting to mark the exact location of stones present in FO1 in Figure 7. Also, it is visible in Figure 1b that there are 2 stones present extremely near to FO2. One is at z = 30km (below FO2) and x is between 9 to 10km and the other is on FO1. It will be interesting if the authors can try to observe and distinguish between these two stones from the strain amplitudes shown in Figure 7.
  We have added vertical lines in Fig 8 (formerly 7) to display the stone locations.

- 25. Page 12 : I request the authors to use numbers instead of bullets to enlist the main points of the development of homogenization theory.
  Done

- 26. I request the authors to add the word 'equation' before referring to any equation in the text to provide better readability to the manuscript.
  Done

- 27. Page 14, Line 327-330: The authors mention that a spatially smaller or larger heterogeneity has a similar amplitude effect on the strain. As per the understanding of the reviewer, the authors, in the present numerical simulation have considered the stones of the same size to represent heterogeneties around FO1 and FO2. It will be extremely interesting to show another set of simulations where the stones of different sizes are cautiously placed near FO1 and FO2 and plot figures similar to the left panels of Figures 3 and 4 and Figure 5. This set of simulations and corresponding figures will provide a better understanding of the statements mentioned in these lines.
  Indeed, it is a non-intuitive behavior. We have added a reference in which this effect is illustrated.

- 28. Page 15, Line 348-350: Kindly briefly describe the method to obtain effective elastic tensor so that the readers can refer to Browaeys and Chevrot (2004) for its details. The added explanation will provide a better understanding to Figure 10 for the readers.
  Browaeys and Chevrot are used only to compute the nearest isotropic elastic tensor $\mathbf{c}^{\text{iso}}$ to an arbitrary tensor $\mathbf{c}$. It is a simple method (and not a homogenization method) and we use it to represent the effective tensor obtained with the homogenization method Capdeville et al. (2010) described in the paper.

- 29. Page 15: Figure 8 caption- Change upper right panel to upper left panel. Also, the upper panels of Figure 8 can be combined to show the velocity in the original media and effective media in the same subplot as the authors have represented density.
  We made two separated plots for V_S and $V_S^{*\text{iso}}$ because the difference of amplitude makes the $V_S^{*\text{iso}}$ difficult to visualize if plotted in a single graph.

- 30. Page 16 : How many iterations are carried out by the authors to get the close match between the simulations in the original media and effective media shown in Figure 10b and 10d.
  It is not clear what is meant by iterations here as it is not an iterative method. There are only 3 steps to obtain the effective traces shown in Fig 11 (formerly 10):
  1. obtain the effective media and the correctors with the homogenization tool presented in section 5.1
  2. run SEM in the effective media (it is a standard SEM run) to obtain Fig 11 a and b
  3. apply the correctors to obtain Fig 11 c and d
- 31. Page 17, Line 388: Please provide appropriate references that show the numerical evaluation of small-scale heterogeneties on DAS measurements.
  This is the point of the paper. To our knowledge, there is no other.
- 32. It will be interesting if the authors try to simulate the 2.7Mw earthquake mentioned in Figure 1 assuming as an effective media and comparing it with the records that they have from the DAS measurements in the original media. This will further support the fact that by circumventing the effort that researchers may have to go through to characterize the heterogeneities in the substructure of the Earth to obtain realistic estimates of strain and rotations, the homogenization method can provide reliable results.
  This is a difficult request because we do not have a good model (at least good enough) of the area. This would nevertheless be possible with limited accuracy and gathering models and geology of the area and then using a 3-D SEM tool. The amount of work required to achieve such a goal makes us acknowledge that we can not comply for now, but it will be for a future work.

  Nevertheless, the requests show a problem in the writing of our paper. Indeed, it indicates that the reviewer thinks that we are trying to model the data shown in section 3, which we are not. The data are just here to illustrate the effect we are proposing to explain.

References

Capdeville, Y. & Marigo, J.-J., 2013. A non-periodic two scale asymptotic method to take account of rough topographies for 2-d elastic wave propagation. *Geophys. J. Int. 192*(1), 163–189.

Dahlen, F. A. & Tromp, J., 1998. *Theoretical Global Seismology*. Princeton University Press. NJ.

Seriani, G. & Priolo, E., 1994. Spectral element method for acoustic wave simulation in heterogeneous media. *Finite elements in analysis and design 16*(3-4), 337–348.