Non-peer reviewed manuscript submitted to SEISMICA

Supporting Information for "SeisMIC - an Open Source Python Toolset to Compute Velocity Changes from Ambient Seismic Noise"

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1 Introduction

In this supporting information, we elaborate on the approximation of the stretching factor implemented in SeisMIC. We show a short derivation in section 2.

In section 3, we present selected results of the spatial inversion procedure to illustrate the effect of (1) subdividing the coda to obtain several independent velocity change estimates (hereafter referred to as coda splitting) and (2) adding information from autocorrelations and cross-station correlations to the analysis. To this end, we provide several examples using different station configurations and damping parameters. For a thorough discussion of the algorithm, please refer to the main manuscript. The reader can reproduce and extend the shown results using the `spatial.ipynb` Jupyter notebook provided in the digital supplement.

For the coda splitting, we split the total of the used coda window into three equally long subwindows and obtain one sensitivity kernel for each subwindow. That is when exploiting the coda splitting, we compute separate sensitivity kernels for lapse times from 14 to 20.66 s, from 20.66 s to 27.33 s, and from 27.33 s to 34 s, whereas without coda splitting, we only obtain one sensitivity kernel representing lag times from 14 s to 34 s. To all forward-modelled dv/v results, we add random noise from a Gaussian distribution with a standard deviation of only 0.05% dv/v to the predicted velocity changes to render the comparison less dependent on the noise level. Otherwise, the procedure for the spatial inversion remains identical to the one described in the main manuscript section 3.5.

Apart from this supporting document, we provide Jupyter notebooks, computing scripts, and the main program, SeisMIC 0.5.3, as a digital supplement. For SeisMIC, however, we strongly encourage the reader to obtain the code's latest version, for example, from GitHub. For details about where to retrieve these additional supplements, please refer to the *data and code availability* section in the main manuscript.

2 Derivation of the Stretching

A problem with the commonly used definition of a velocity change dv/v is that it is neither additive nor reversible, i.e. $dv/v_{A\to B} \neq -dv/v_{B\to A}$ because the reference velocity of the two measurements may be different and the commonly

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used equation $dv/v = -dt/t$ is an approximation for small dv/v. Even though the error is small, we can reduce it further with a different definition of the velocity change. This definition is especially useful for smoothing, averaging, or otherwise manipulating velocity change time series as, in contrast to the common definition, it ensures linearity (see below).

Assuming infinite frequency (i.e., ray theory), the travel time $t(\mathbf{x})$ of a wave travelling from point $\mathbf{x}_1$ to $\mathbf{x}_2$ through any medium is given by:

$$t(\mathbf{x}) = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \frac{1}{v(\mathbf{x})} d\mathbf{x} \tag{1}$$

where $v(\mathbf{x})$ is the velocity at point $\mathbf{x}$. If we assume a medium with a homogeneous velocity, i.e., $v(\mathbf{x}) = v$, equation 1 simplifies to:

$$t = \frac{\Delta x}{v} \tag{2}$$

where $\Delta x = ||\mathbf{x_1} - \mathbf{x_2}||$ is the Euclidean distance which remains constant throughout. In interferometry, we examine the case where the velocity $v(t)$ is variable with time. To quantify the change in velocity, we compare the velocity of a reference state $\tilde{v} = \Delta x/\tilde{t}$ with the velocity $v' = \Delta x/t'$ in a perturbed state. Due to the constant Δx, we obtain from equation 2:

$$\frac{\tilde{t}}{t'} = \frac{v'}{\tilde{v}} = \xi \tag{3}$$

defining the stretching factor ξ.

We seek a transformation

$$\mathcal{S}(\kappa) : \tilde{t} \mapsto t' = \frac{1}{\xi}\tilde{t}$$

that maps the original travel times $\tilde{t}$ of the seismic waves through the unperturbed medium to the travel times t' through the perturbed medium. This transformation shall have the following property:

$$\mathcal{S}(\kappa_1)\mathcal{S}(\kappa_2) = \mathcal{S}(\kappa_1 + \kappa_2)$$

to ensure that stretching is additive and reversible. We choose

$$\xi = e^{\kappa}$$

to calculate the stretching factor. We satisfy the above requirement for any combination of reference and perturbed states since $e^{\kappa_1} * e^{\kappa_2} = e^{\kappa_1+\kappa_2}$. In SeisMIC, we implement $\xi = e^{\kappa}$ to guarantee the linearity in the processing. To interpret measurements in the usual way as fractional velocity change, we use the approximation

$$\frac{dv}{v} = \frac{v' - \tilde{v}}{\tilde{v}} = \xi - 1 \approx \kappa$$

.

Utilising $\xi = 1 + \frac{dv}{v}$ for processing introduces nonlinearities and a dependency on the reference state. We note that this effect is not limited to measurements of the velocity change with the stretching method. Any manipulation of $\frac{dv}{v}$ is affected and can be improved by working with $\kappa = \log(v'/\tilde{v})$.

3 Additional Results of the Synthetic Spatial Imaging

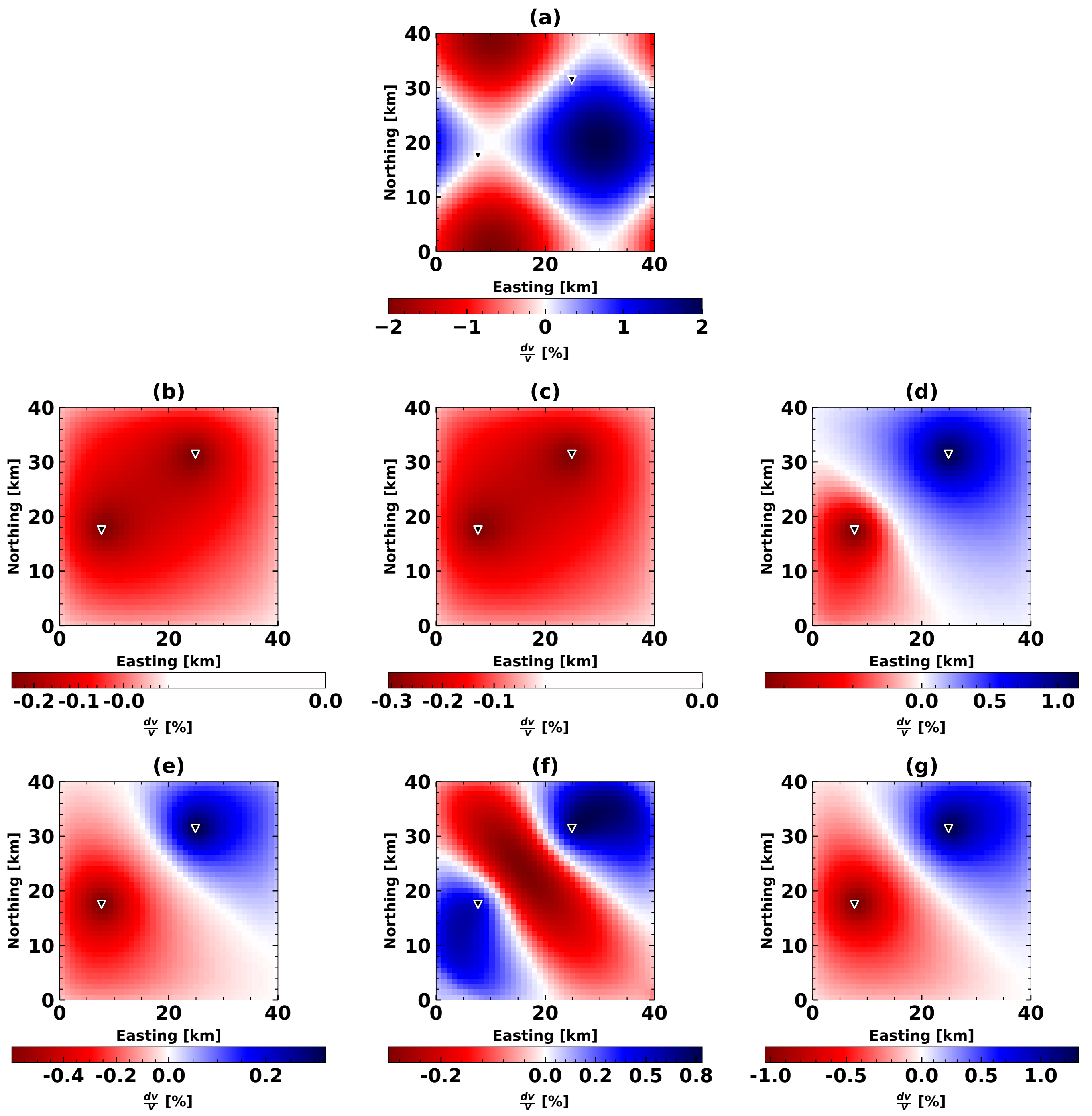

Figure 1 Examples of the spatial inversion using data from two stations, a model variance $\sigma_m = 0.5\ \frac{\mathrm{km}}{\mathrm{km}^2}$, and a correlation length $\lambda = 2$ km. **(a)** The synthetic velocity model and station configuration used. **(b)** Result of the spatial inversion using only cross-correlations and a single lapse time window. **(c)** Result of the spatial inversion using only cross-correlations and three lapse time windows. **(d)** Result of the spatial inversion using only auto-correlations and a single lapse time window. **(e)** Result of the spatial inversion using only auto-correlations and three lapse time windows. **(f)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window. **(g)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window.

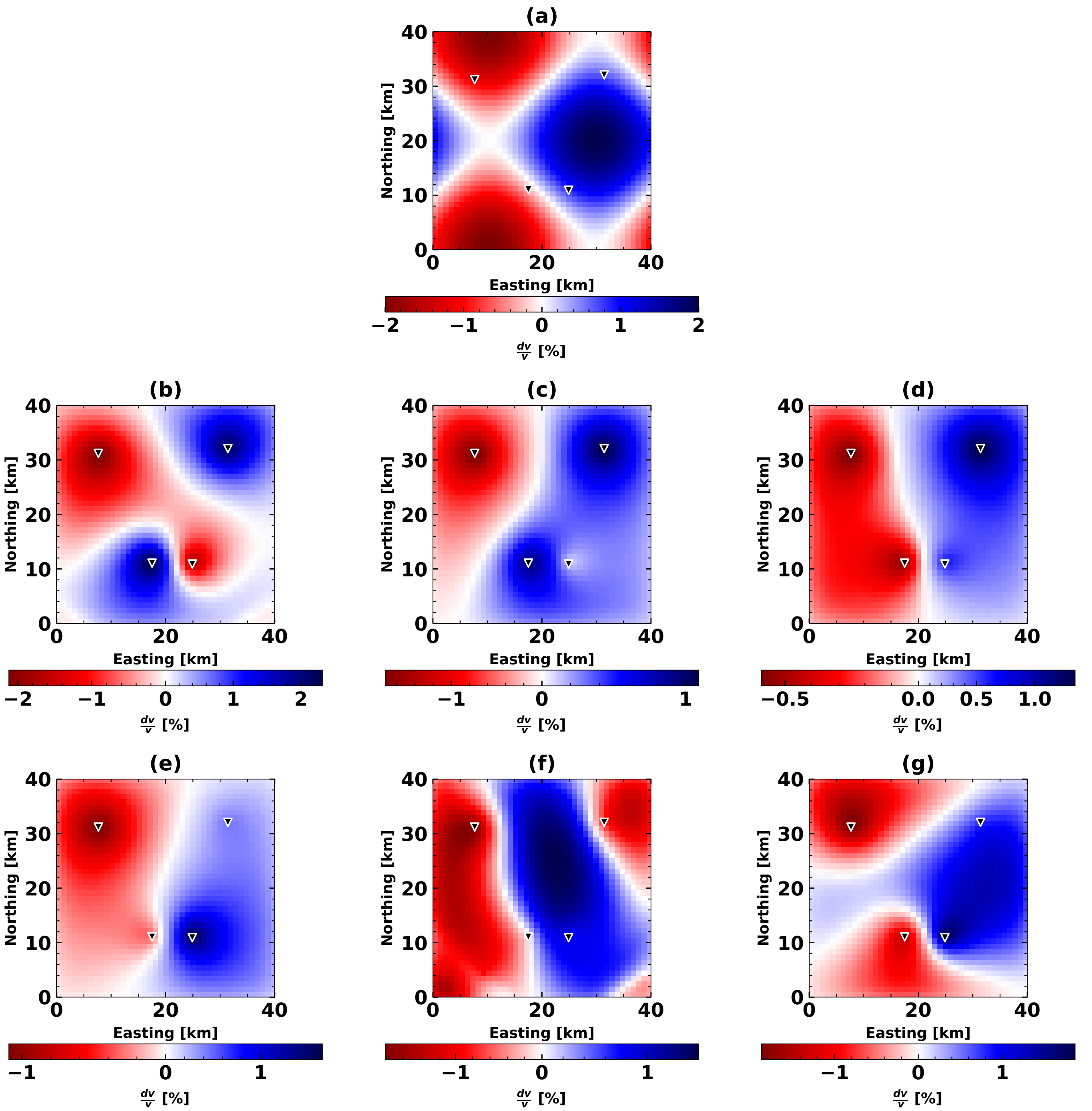

Figure 2 Examples of the spatial inversion using data from four stations, a model variance $\sigma_m = 0.25\,\frac{\mathrm{km}}{\mathrm{km}^2}$, and a correlation length $\lambda = 2$ km. **(a)** The synthetic velocity model and station configuration used. **(b)** Result of the spatial inversion using only cross-correlations and a single lapse time window. **(c)** Result of the spatial inversion using only cross-correlations and three lapse time windows. **(d)** Result of the spatial inversion using only auto-correlations and a single lapse time window. **(e)** Result of the spatial inversion using only auto-correlations and three lapse time windows. **(f)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window. **(g)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window.

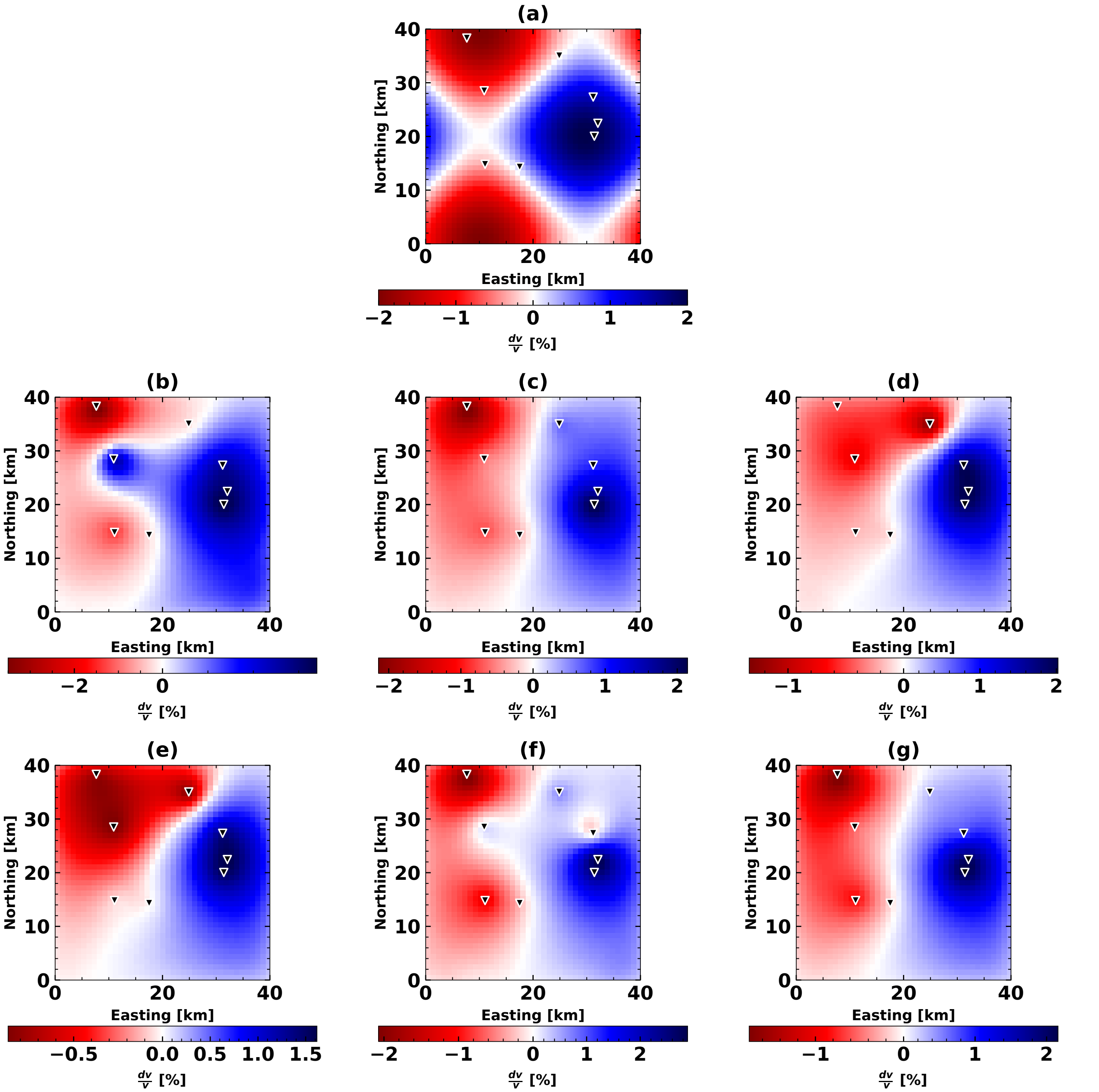

Figure 3 Examples of the spatial inversion using data from eight stations, a model variance $\sigma_m = 0.05\ \frac{\text{km}}{\text{km}^2}$, and a correlation length $\lambda = 2$ km. **(a)** The synthetic velocity model and station configuration used. **(b)** Result of the spatial inversion using only cross-correlations and a single lapse time window. **(c)** Result of the spatial inversion using only cross-correlations and three lapse time windows. **(d)** Result of the spatial inversion using only auto-correlations and a single lapse time window. **(e)** Result of the spatial inversion using only auto-correlations and three lapse time windows. **(f)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window. **(g)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window.

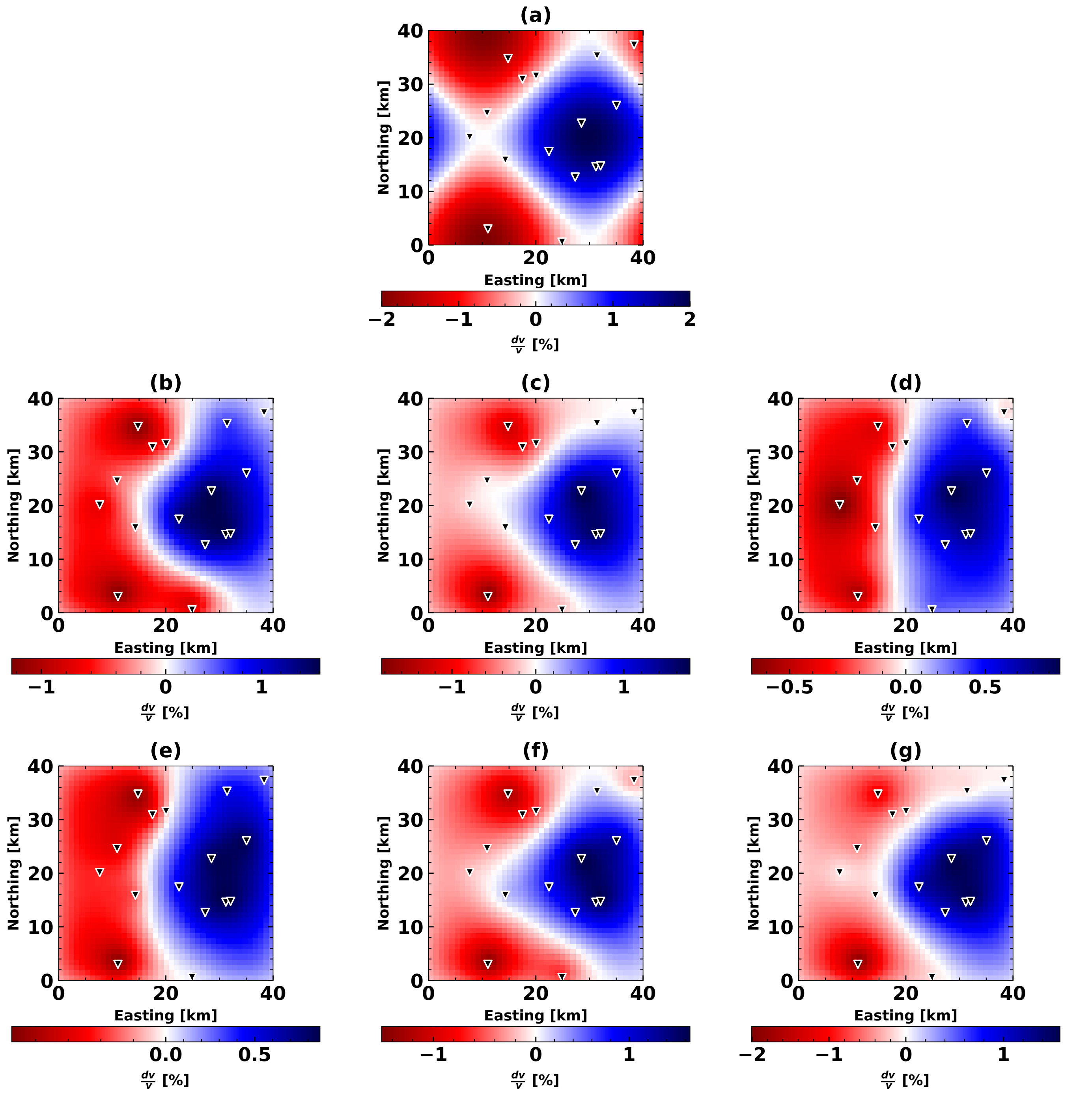

Figure 4 Examples of the spatial inversion using data from 16 stations, a model variance $\sigma_m = 0.02\ \frac{\text{km}}{\text{km}^2}$, and a correlation length $\lambda = 2$ km. **(a)** The synthetic velocity model and station configuration used. **(b)** Result of the spatial inversion using only cross-correlations and a single lapse time window. **(c)** Result of the spatial inversion using only cross-correlations and three lapse time windows. **(d)** Result of the spatial inversion using only auto-correlations and a single lapse time window. **(e)** Result of the spatial inversion using only auto-correlations and three lapse time windows. **(f)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window. **(g)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window.

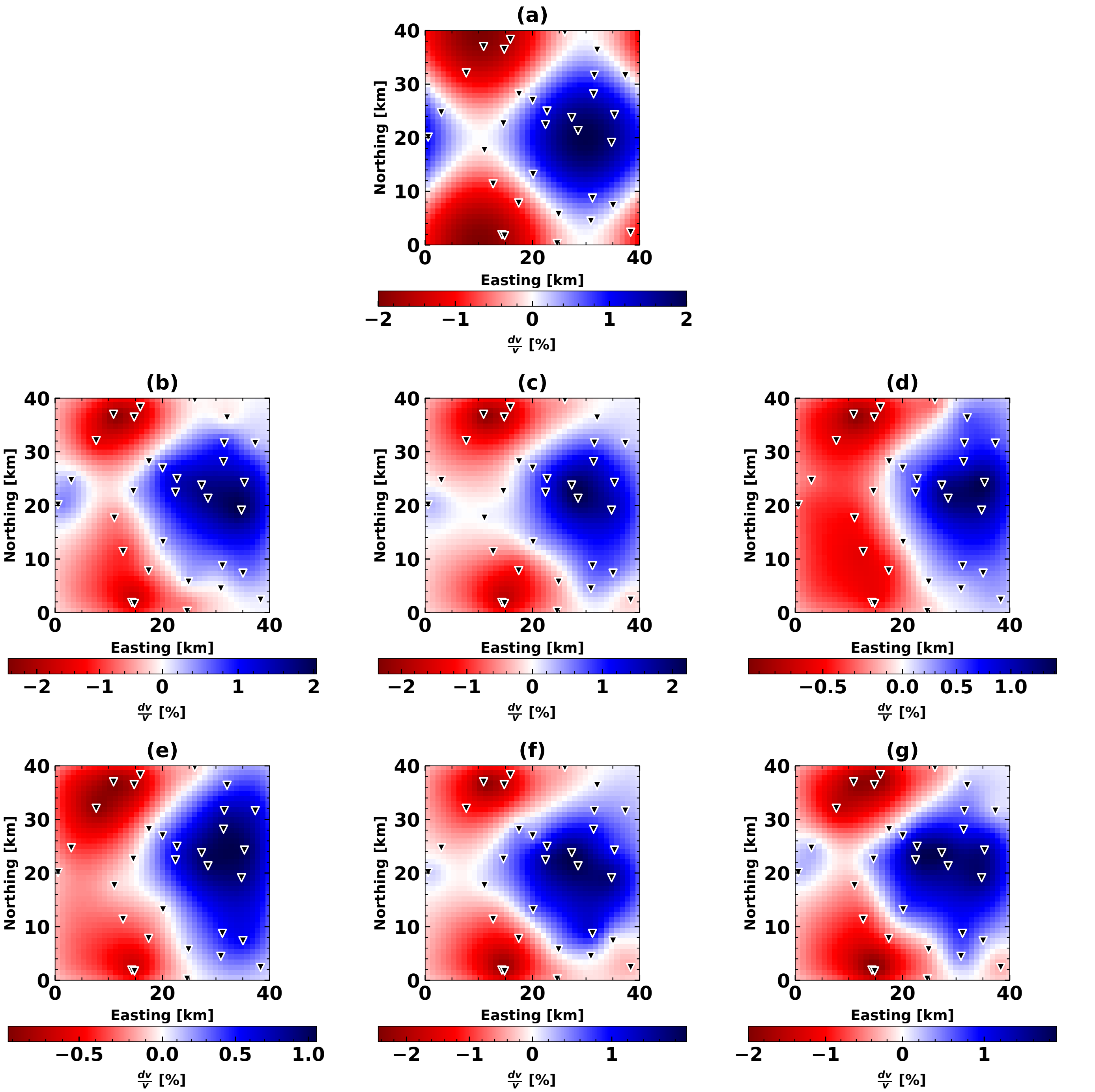

Figure 5 Examples of the spatial inversion using data from 32 stations a model variance $\sigma_m = 0.05 \frac{\mathrm{km}}{\mathrm{km}^2}$ and a correlation length $\lambda = 2$ km. **(a)** The synthetic velocity model and station configuration used. **(b)** Result of the spatial inversion using only cross-correlations and a single lapse time window. **(c)** Result of the spatial inversion using only cross-correlations and three lapse time windows. **(d)** Result of the spatial inversion using only auto-correlations and a single lapse time window. **(e)** Result of the spatial inversion using only auto-correlations and three lapse time windows. **(f)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window. **(g)** Result of the spatial inversion from cross-correlations and auto correlations using a single lapse time window.