11 September 2024

Dear Editor,

We would like to thank both reviewers for making an effort to improve our work and for their thorough yet constructive comments. We address these below, where you find our response coloured and in italics, followed by a summary of all changes applied (p. 5).

Best regards,

Katrin Löer & Claudia Finger

Reviewer A:

Some of my suggestions for the manuscript are given below, and I hope that the authors will find them useful.

1. Ideally, the results of equations 5 and 6 are theoretically consistent. For a real ambient noise field, the cross-correlation function is not symmetric, and the CSDM may have an imaginary part. Equation 5 is likely complex, whereas Equation 6 is positive because the square of the modules is taken as the output there. The authors may wish to comment on this point.

   Thank you for taking the time to investigate these equations. In theory, the outcome of equation 5 is real, just as the outcome of equation 6. The operations applied effectively lead to a summation of complex numbers over station pairs. Because we consider station pair $s_1 s_2^*$ as well as $s_2 s_1^*$ we end up summing complex conjugates resulting in a real value: $e^{i\phi} + e^{-i\phi} = 2\cos\phi$. In reality, there might occur numerical errors, which is why we take only the real part of both equations in the code. Note that equations 5 and 6 are performed in the frequency domain and solved separately for each frequency, hence, time- or frequency-dependent asymmetries are not relevant here. The cross-spectral density matrix S is symmetric.

   We added a sentence in the manuscript to clarify that the beam resonse B is real valued and a function of frequency and wavenumber for all three equations 4, 5, and 6 (p. 3, l. 91).

2. Around equations 7,8,9, it would be good to give a schematic of the coordinate system, which shows the variables occurred in the equations and the angular rotation conventions, so that readers will be able to get a clearer idea of the coordinate system used by the toolbox. This is very important to investigate the

beamformer output since different researchers may use different coordinate conventions.

We agree that this would indeed be insightful for the reader! We have added a figure visualising the coordinate system used for the three rotation angles (new Fig. 1, p. 5) and an example of the particle motion ellipse for a Rayleigh wave (new Fig. 2, p. 6).

3. Please make sure equation 8 is correct and agree with the coordinate conventions. Exchange the first and third line?

   Thank you for thoroughly reviewing these equations. We believe that equation 8 is correct, which should become clear from the new figure of the coordinate system (see comment 2).

4. Line 127 "Disregarding the azimuth rotation (Rx)(Rz?), or assuming pha=0". You mean ksi=0? If phi=0, there should be Ry.Rx (rather than Rz.Ry) in equation 10? Please make sure equaitons 7,8,9 and 10 are correct. And more importantly, agree with the given coordinate conventions.

   *We thank the reviewer for spotting this negligence. The matrices involved in creating matrix **R** (eq. 10) are indeed $\mathbf{R}_y$ and $\mathbf{R}_x$ (not $\mathbf{R}_z$). We have corrected this in the manuscript (p. 5, l. 130 and eq. 10).*

5. I have to say it was not an easy task to understand the text from line 130 to 155. On the one hand, the analysis of polarization for surface waves is inherently complex. On the other hand, some details are seeming to omitted in the manuscript. In line 135, a and b are vectors? Bold style? Would you please add some comments on vectors a and b? For example, how do they relate to the polarization ellipse of a Rayleigh wave? Are they the long and short half axes of the ellipse?

   *We appreciate the reviewer's effort to improve the clarity of this part. It is correct that **a** and **b** are vectors and should be bold in eq. 12 – this has now been changed. It is also correct that **a** and **b** are related to the ellipse representing the normalised horizontal (**a**) and vertical (**b**) half axis, respectively. We have added this explanation to the manuscript (p. 5, l. 137) and in the new Fig. 2 (p. 6).*

6. When one search for the dip (theta), as shown in Table 1, it is always assumed to be 90 degree for the Rayleigh wave? Right? Would you please give the expression for c(p) occurred in w(k,p) in equation 14? as a function of polarization parameters.

   *We apologies this was unnessecarily confusing in the manuscript: the phase shift vector **Z** in equation 11 and in Table 1 is in fact equivalent to the phase shift vector* *called **c**(p) in equation 14 and the following text. We have changed the variable naming to make both the same, which we hope clarifies the issue.*

7. For the example shown in Figure 1 based on the synthetic data, it would be good to give some details on the parameters used to synthesize the data and the contribution of different wave types. What does the synthesized data look like in the time domain? Does it contain only retrograde Rayleigh waves? Or are there other wave types also included? What does mean the color scale in Figure 1b? Does it relate to parameters in Table 1 of the Supplementary Material? Why does it look different from the figure in the Supplementary Material?

   Thanks for highlighting the discrepency between Figure 1 and the figures in the Supplementary Material. We decided to use the colorbar as in the Supplementary Material in all Figures, showing the polarisation index as now explained in Table 1. We have added an explanation to the caption of Fig. 1 (now Fig. 3). In the Supplementary Material we now use the same colormap as in the manuscript.

   The synthetic data was created with a finite-difference wave-propagation modelling code from a single source in a homogeneous half-space with a free surface. The complete wavefield thus contains more waves than just Rayleigh, however, in the time window analysed here the Rayleigh wave was clearly recognised as the dominant contribution. We clarify this in the caption of Fig. 3.

8. Line 183, the address of the LINK?

   Thanks – has been added in the final manuscript (p. 7, ll. 186 ff.).

9. Line 399, “In Fig. XXX?”

   Apologies, this should have been Fig. 9 (new Fig. 11) – corrected (p. 17, l. 403).

10. In Figures 6 ,7 and 8, the text is too small to read. It is not easy to seen the white dotted line in Fig. 7, similarly for the white “cross” in Figure 1a

    Thanks for pointing this out. We have increased marker size, line width and font size in the respective figures (see new Figures 3 and 8-10).

11. For the naming rules of the file name, XN means station array, yyyy means the year, ddd means day of the year, fff means frequency, etc., which appear many times in the text, such as line 197, 219, 262, etc., it is sufficient to state the naming rules in a separate place.

    Indeed a lot of redundancy. We provide these rules in section 3.1 (p. 9, ll. 202 ff.) and refer to them when required.

Reviewer B

Some suggestions for improvements.

- A styling suggestion: I would not use “you” to directly address the article to the reader. Maybe the authors can rewrite in a neutral form

  Thanks for the feedback. The direct form was deliberately chosen for the part that concerns the instructions for using the code, writing it more like a tutorial that should be easy to follow. We find that using a neutral or passive form makes it more difficult to read but we would appreciate the opinion of the editor in this matter and will be happy to change it if that should be the preferred style.

- The (LINK, L183) to the github will probably be added in the final version (https://github.com/cl-finger/B3Ampy, https://github.com/katrinloer/B3AM)

  Yes, links to both the MATLAB and Python version have now been added to the manuscript (see p. 7, ll. 186 ff.).

- L171; analaysis -> analysis

  Thanks for spotting the typo – we’ve corrected it!

Summary of changes:

Figures:

- Figure added to explain coordinate system of rotation matrices (new Fig. 1)
- Figure added to explain the role of vectors a and b as the half axes of the Rayleigh wave particle motion ellipse (new Fig. 3)
- Marker size changed in new Fig. 3
- Line width and y-axis changed in new Fig. 9
- Font size changed in Figs. 3, 8, 9, 10
- Colormap changed for figures in Supplementary Material (now equivalent to Fig. 3)
- Description of synthetic data added to caption of new Fig. 3

Main text:

- Explanation added for Eq. 4 (p. 3)
- Mistake corrected concerning rotation matrices on p. 5, l. 130 and Eq. 10
- Explanation added for vectors **a** and **b**, changed to bold symbols (p. 5, l. 137 and in the new Fig. 2, p. 6).
- New variable name (z) introduced to make Z and c equivalent (Eq. 14, Table 1, p. 6 ll. 153 ff.)
- Typo corrected ("analaysis", p. 7, l. 175)
- Links to GitHub repositories added (p. 7, ll. 186 ff.)
- Redundant description reduced and summarised (p. 9, ll. 202 ff.)
- Reference to figure added (p. 17, l. 403)
- Acknowledgements to anonymous reviewerers added (p. 19, l. 463)

Supplementary Material:

- Page numbers added
- Mistake in Table corrected (now showing precise values, rather than range, for the polarisation index of each simulated wave; p. 1, Table 1)
- Colormap of figures made consistent with that used in manuscript (pp. 2-6)