Supplementary Material for:

Source characterization of the 20th May 2024 M_D 4.4 Campi Flegrei caldera earthquake through a joint source-propagation probabilistic inversion

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Text S1. Pre-processing and processing parameters

We select a 3-second S-wave time window for the inversion, starting 0.1 s before the S-wave arrival time. We then follow the signal and spectrum pre-processing described in Supino et al. (2019). The main steps are the following: (1) We remove constant and linear trend from the signal, and then apply a Hann-function tapering on the first and last 5% of the data. (2) We obtain the S-wave spectrum computing FFT of the pre-processed signal. The displacement amplitude spectrum (Ω_0) is smoothed with a 5-point moving average filter, and finally converted to seismic moment units (M_0) according to the following equation (Aki & Richards, 1980):

$$\mathrm{M}_0 = \Omega_0 \cdot \frac{1}{\xi} = \Omega_0 \cdot \frac{4\pi \cdot \rho \cdot {v_S}^3 \cdot r_H}{R_{\theta\varphi} \cdot F_S} \qquad \text{(S1)}$$

where ρ is the propagation medium density, v_S is the S-wave velocity, r_H is the hypocentral distance, $R_{\theta\varphi}$ is the radiation pattern coefficient and F_S is the free-surface coefficient.

We used ρ = 2500 kg m^{-3} (Judenherc & Zollo, 2004), v_S = 1685 m s^{-1} (Orsi et al., 1999), $R_{\theta\varphi}$ = 0.63 (Aki & Richards, 1980; Boore & Boatwright, 1984), F_S = 2. The values assigned to the parameters are similar to those considered by Petrosino et al. (2008) for the study region.

The hypocentral distance r_H has been computed for each station assuming the INGV-OV event location (latitude = 40.8277°N, longitude = 14.138°E, depth = 2.6 km).

The frequency band used for the inversion is 0.2 – 40 Hz. For each station we invert a single spectrum $\log \tilde{u}(f)$, with $\tilde{u}(f)$ geometric mean of the displacement amplitude spectra of the two horizontal components. The forward operator used for the inversion is the same as defined in Supino et al. (2019), eq. (4):

$$g(\boldsymbol{m} = (M_0, f_c, \gamma, Q'); f) = \log M_0 - \log\left[1 + \left(\frac{f}{f_c}\right)^{\gamma}\right] + \log \xi - \log e \cdot \pi \frac{r_H}{v_S} f \cdot Q' \qquad \text{(S2)}$$

with M_0 (seismic moment), f_c (corner frequency), γ (high-frequency decay exponent) source parameters of a generalized Brune (1970) spectral model, $Q' = {}^{1}/_{Q} + {}^{k_0 \cdot v_S}/_{r_H}$ with Q anelastic attenuation quality factor and k_0 site-dependent attenuation term (Anderson & Hough, 1984), and ξ, v_S, r_H as in eq. S1.

We do not consider a site amplification term in the forward operator, since the inversion is performed on a single event while a set of events is needed to constrain site amplification. Nevertheless, the large number of stations used to obtain the averaged event source parameters is expected to mitigate possible site amplification effects, if any.

The probabilistic framework used to estimate the joint a-posteriori PDF $\sigma(\boldsymbol{m})$ is the same as defined in Supino et al. (2019), based on the work by Tarantola (2005), with the hypothesis of a uniform a-priori PDF over the model space, and of modelization and data uncertainties normally distributed (see Section 3.1 and eqs. 5,6 in Supino et al. (2019) for details).

Test S2. P-waves inversion

We inverted P-waves using the same methodology and parameters described for S-wave inversions, changing the following values according to the selected phase: v_P = 3000 m s^{-1} (Orsi et al., 1999), $R_{\theta\varphi}$ = 0.52 (Aki & Richards, 1980; Boore & Boatwright, 1984), and inverting the vertical component spectrum.

We obtained $M_w = 3.55 \pm 0.06$ selecting a 1-second time window, and $M_w = 3.79 \pm 0.06$ selecting a 1.5-second time window.

As we show in Figure S1, with the 1.5-second time window we selected part of the S-wave, while the 1-second time window was not enough to include the whole P-phase. This is the observational reason why the two M_w values obtained for the two P-wave inversions probably represent an underestimation and an overestimation of the moment magnitude for the earthquake.

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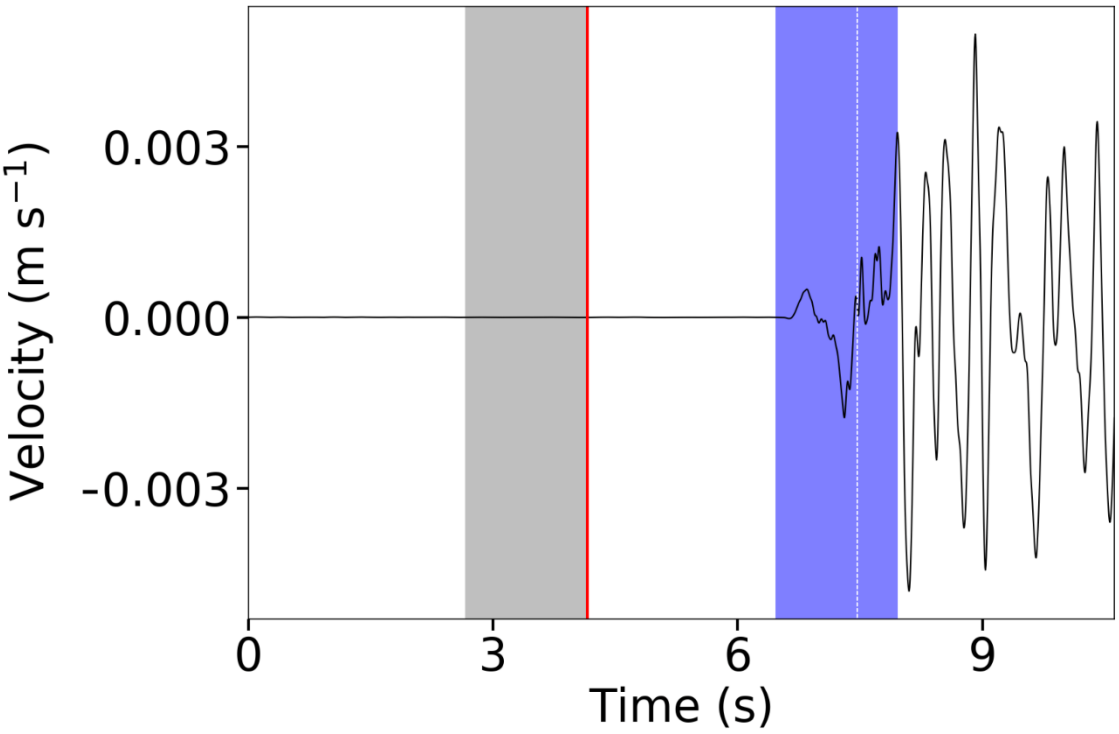

Figure S1. 1.5-second P-wave time window (blue rectangle). The red vertical line shows the origin time of the earthquake. The white dotted vertical line indicates 1-second after the start of the time window. Station CMIS, vertical component.

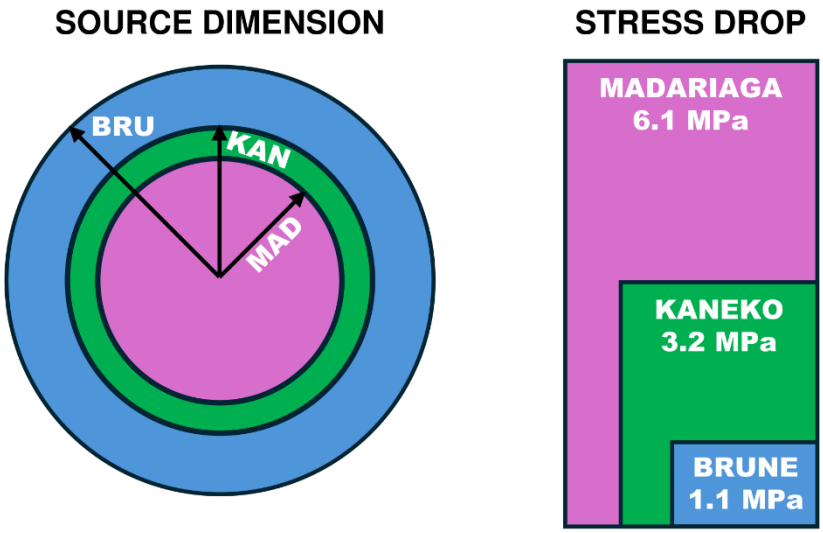

Figure S2. Visual comparison between source dimension and stress drop estimated assuming Brune (1970), Kaneko and Shearer (2014) and Madariaga (1976) model.