Response to comments from reviewers

Seismica manuscript `1433`:

Sparse fault representation based on moment tensors interpolation

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Letter to Editor

Dear Prof. Xu:

Thank you very much for overseeing my manuscript. In response to comments from the two anonymous reviewers, I have made several updates, the most notable being:

1. Modification of Figure 9. Reviewer C has prompted for a modification to improve Figure 9 readability. I have reworked the color scheme to improve contrast, and color-coded the moment tensor by tensor-density, in order to better convey all the displayed metrics.

2. I have added discussion points brought forward by the two reviewers, related to quantitative computational comparison, direct angular-interpolation vs. eigen-decomposition based interpolation, and applicability beyond simple fault geometry.

3. Update of the accompanying `Zenodo` collection, to reflect a minor algorithmic change that should better account for edge-cases in the proposed interpolation method, in the script to reproduce Figures 3-6 and Figure 7. This does not entail a modification of these figures, as they did not contain the aforementioned edge-cases.

I greatly appreciate the time that the reviewers and editor have put into the process of helping to improve this manuscript. I have addressed all remaining points, and am looking forward to hearing back from you.

Thank you for your consideration.

Sincerely,
Julien Thurin

Response to Anonymous reviewer C

The paper proposes an approach to represent finite fault models more efficiently by leveraging a quaternion-based rotation of eigenvectors in moment tensor interpolation. This methodology aims to reduce the degrees of freedom typically required for large-scale finite fault models while approximating complex fault slip behavior. The mathematical formulations are likely correct, and the approach effectively aligns with concepts like Kagan angle rotation to capture smooth transitions in eigenvector orientations, which is an improvement over traditional summation methods. The reduction in degrees of freedom is promising, especially for extensive finite fault models. This method allows a more streamlined model representation without sacrificing much accuracy, as demonstrated in the approximation of the USGS finite fault model for the 2024 Noto earthquake. However, I have some questions for author to clarify/address.

1. In cases of pure double-couple sources, which are common in finite fault slip models, strike/dip/rake interpolation may offer a more natural alternative for capturing smooth focal mechanism rotations. While this approach may not sample the moment tensor space uniformly (and is sometimes not shape-preserving), it has an advantage over direct moment tensor summation by avoiding the introduction of non-double-couple components and allowing for smoothly varying geometric parameters. When strike and dip are fixed in finite fault models, adjusting only the rake angle can represent all possible slip directions and fit the data effectively—additionally, this approach simplifies to a linear problem. The authors should clarify the advantages and potential limitations of rotation-based interpolation in comparison to strike/dip/rake-based interpolation, highlighting where each method is most beneficial.

   RESPONSE: Thank you for bringing up this important point. When the fault plane is fixed, interpolation based on rake angle alone is indeed very efficient: it is the simplest method both in terms of computation and implementation and probably the ideal choice if you want to closely stick to a given fault plane, as it will guarantee a rotation about the fault normal vector (and it is also equivalent to direct linear combination for the most part, except for a few edge cases). When strike and dip are fixed, quaternion-based interpolation is equivalent, but this only holds true for small angular differences: when performing the rotation between the -90° and 90° rake angles, the shortest path is actually a rotation of 90° rotation about the Null axis (2nd eigenvector) of the moment tensor (which is equivalent to a sign-flip), instead of a long rotation of 180° along the fault-normal (Figures R1 and R2).

   When angle differences are smaller than 90°, then both methods are fully equivalent (Figures R3 and R4).

   Another disadvantage of angular interpolation is that it does not guarantee smooth interpolation when all three angles are considered, especially when considering more than two sources and adding significant angular discontinuities (large strike and rake differences, for instance). The quaternion-based interpolation provides much smoother tensor fields (it reduces to the rotation at a constant rate over a single axis, while the angular rotation has three different, non-linear rotational velocities, Figures R5 and R6).

   The advantage of the quaternion-based approach is also apparent when considering potential applications beyond the (admittedly) simple proof of concept – i.e., when considering non-double couple or when removing the fault plane out of the equation to perform slip and geometric inversion, as guaranteeing shorter rotation would allow to develop smoother parametric surfaces. In order to convey this advantage, the following has been added to section 3:

   line 134 - "*For pure double-couple sources, strike, dip, and rake angles offer a natural parameterization for describing focal mechanisms and can also serve as a basis for interpolation.*

When the fault plane is fixed, interpolating over rake angles alone provides a straightforward, source-preserving method. However, It has notable limitations, particularly when considering full moment tensor rotation: It may produce a sub-optimal interpolation path when angular distances are large or if differential rotation between all three angles is not scaled properly, leading to inconsistent rotational velocities. Thus, angular interpolation fails to generalize to cases where the fault plane is unknown or if non-double-couple components are present, requiring a more flexible approach capable of handling arbitrary source mechanisms."

(Figures are provided at the end of this letter.)

2. Although the proposed method reduces unknowns compared to traditional finite fault models, the paper could benefit from a more explicit quantitative comparison. For instance, a finite fault model with at least 315 fault segments might involve four parameters per segment (rake, rise time, rupture time, and moment), amounting to at least 1,260 unknowns. In contrast, the 9 key tensors in this approach involve only 81 unknowns, assuming fault geometry is predefined. Providing such comparisons can help readers appreciate the efficiency gains.

   RESPONSE: The initial rationale for a number-of-source-based comparison is that the number of unknowns might change depending on the assumptions taken when inverting for a finite fault model. The main goal of this proof-of-concept was to show that the tensor interpolation allows for a representation akin to a "PCA for slip models," which I think we should lean toward in modernizing inversion workflows, such that we can exploit the inherent low-dimensionality of the problem. I have added the following paragraph in section 4.1 to discuss the efficiency gains:

   line 271 - "*Considering the USGS NEIC model is discretized over 315 sub-faults, the number of unknown parameters in the original model amounts to 1260 (for rake angle, rise-time, rupture time, and moment"). By contrast, our approach reduces the number of free parameters to 81, using nine key tensors, each defined by nine parameters. While the temporal aspect has been left out for the sake of simplicity, adding the rupture time and rise time would raise the approximation to 99 parameters, which is still a significant improvement over the original parameterization. We also note that strike and dip angles could have been left out of the application to further reduce the number of unknowns. They have been kept as free parameters to demonstrate the potential for more general applications, particularly for scenarios where fault geometry is unknown or where tensors are projected onto non-planar parametric surfaces.*"

3. The interpolation method assumes a fixed fault geometry, which contradicts the flexibility that the reduced number of moment tensors could theoretically provide. If strike and dip for key tensors are not fixed, the method might unintentionally produce fault geometries with arbitrary patterns, such as en échelon or orthogonal ruptures. In other words, strike and dip for key tensors should be consistent with fault plane strike and dip. It would be valuable for the authors to discuss how this constraint impacts the model's applicability and any potential adjustments to address such issues.

   RESPONSE: Currently, the method relies on a two-dimensional manifold on which the interpolation weights are projected (here, the Gaussian-support functions are projected over a 2D linear fault plane, identical to the original USGS-NEIC solution). However, nothing prevents using curvilinear surfaces defined via splines (such as Non-uniform rational B-splines), as they could still be projected in a 2D subspace where the interpolation could be computed. Having access to the moment tensors' positions in space and their respective eigenframe would make it accessible to generate those more complex surfaces parametrically using splines. To make things explicit, I have added the following modification to the discussion (which already hints at the potential of application beyond a simple fault geometry):

line 338 - "*While we have made the choice of simplicity to present this parameterization and relied on a pre-existing 2D plane, this parameterization provides a solid basis for an extension to non-planar faults: each key centroid contains important information about geometry, encoded in the eigenframe of each tensor (and in turn, its fault planes).*"

4. I note that the authors employ a black-box optimization to approximate the slip model using a specified number of key tensors. However, a potential issue arises in managing the ambiguity of the rotation path between key tensors, especially when these tensors are nearly complementary. This concern is particularly evident in Figure 7, where the radiation patterns of the middle and right tensors are complementary. Visual inspection suggests a rotation angle exceeding 90 degrees, which could lead to non-unique rotation paths. Could the authors clarify how this ambiguity is addressed in their approach?

   RESPONSE: Thanks for this remark. Some of the tensors in Figure 7 are indeed nearly opposite in sign, which could lead to ambiguity. Strictly speaking, rotation ambiguity only arises with edge cases such as perfectly opposite tensors (for instance, a trace of [-1,0,1] and [1,0,-1]), where two equally valid 90º rotations exist. While they exist, these ambiguities are also very singular points of the moment tensor space, and it is unlikely that they would be encountered in practical applications. In other cases where the rotation is well defined, using quaternions is practical to lift the ambiguity, as for a unit quaternion q encoding a specific rotation, the quaternion 1 - q performs the opposite (long-path) rotation. By ensuring we always follow the shortest rotation, there should not be any issues in performing interpolation.

   If no external constraint exists on the fault geometry, the best we can do is ensure that we take the shortest rotational path to produce a smooth source model. After some additional tests on pathological tensors, we introduced a slight revision for section 3 that prevents suboptimal rotations for some edge cases that were not previously tested (and which would likely not occur in a real-case scenario application).

   line 161 - "*This can be done by generating all possible permutations of the interpolated tensors with respect to a "reference tensor," such that we can select the right-handed eigenvector-directions that minimize the rotational distance between equivalent eigenframes (amongst a set of 4 opposite-sign right-handed eigenvector combination). Right-handedness is defined as a positive determinant of the eigenvector matrix.*"

   The accompanying scripts on Zenodo have also been updated with the latest algorithmic strategy which implies testing out eigenvector directions permutations. This modification does not changes the result of the figures in the paper, but should prevent unwanted behaviour on some edge cases.

5. Figure 9: The color scale for the normalized key tensor density lacks clarity; only blue contours are readily distinguishable. Improving the color scheme would enhance the figure's interpretability.

   RESPONSE: Figure 9 has been modified: Colored crosses have been shifted to plain black, contour line transparency removed for better contrast, and the moment tensor solutions have been colored according to the contour line values.

6. Figure 10: Each sub-panel should have its own subtitle. Better separated with (a) (b) and (c).

   RESPONSE: Thanks for the suggestion, this comment has been address by adding (a), (b) and (c) above each subplot panel.

7. The introduction should include a reference to some recent subevent inversion studies on complex multi-fault ruptures, as they provide better support on the need of new representations compared to commonly used finite-fault schemes. e.g. Jia, Z., Shen, Z., Zhan, Z., Li, C., Peng, Z. and Gurnis, M., 2020. The 2018 Fiji Mw 8.2 and 7.9 deep earthquakes: One doublet in two slabs. Earth and Planetary Science Letters, 531, p.115997. Jia, Z., Wang, X. and Zhan, Z., 2020. Multifault models of the 2019 Ridgecrest sequence highlight complementary slip and fault junction instability. Geophysical Research Letters, 47(17), p.e2020GL089802. Kagan (1991) should be cited, as this methodology is closely related to the concepts in quaternion rotation and eigenvector manipulation used in the paper. Kagan, Y.Y., 1991. 3-D rotation of double-couple earthquake sources. Geophysical Journal International, 106(3), pp.709-716.

   RESPONSE: The aforementioned references are indeed very much in line with the topic of the manuscript, and have been added in the introduction.

Response to Anonymous reviewer D

This manuscript is very thoughtful and well written. The text is clear and largely devoid of minor errors/typos. I enthusiastically recommend it for publication with Seismica. Below, I mention a few aspects for consideration by the author that I think could strengthen the paper.

1. Given that the USGS Model is available rapidly and this proposed method is geared toward rapid response, I understand why the author used this model as a baseline. However, I would have appreciated some discussion of how it might work relative to some recently published, more complex models. Some, unlike the USGS model, include oppositely-dipping planes and different slip patterns. It would be great if the author could comment on performance relative to such models as well.

   RESPONSE: Thanks a lot for this remark; operational results are indeed one of the targets uses for this method, but nothing is preventing the extension toward either complex pre-defined geometries (assuming we have several fault planes, for instance) or estimating a geometry directly via the information embedded in the moment tensor (similar to what is being done in Shimizu et al. 2020), apart of course from algorithmic considerations. If we assume several fault planes, they can be treated independently, and a tensor field would be computed on each fault plane from the key tensor. The main question is how many tensors are needed and how to distribute them across the different fault planes; otherwise, the method would be compatible with a complex (known) geometry. As mentioned in the discussion, I also think there is a strong potential to develop curved surfaces directly from the moment tensors (as they possess the geometrical information to act as control and tangent points), but there would be a need to build a complete strategy from the ground up on how to construct fault planes (should they blend into one-another using splines, or should they just be intersecting for instance)?

   The following has been added to the discussion in order to clarify its compatibility with fault complexity / multiple fault planes:

line 345 - *"Note that the current parameterization would readily allow consideration of multiple independent fixed fault planes without algorithmic modifications by distributing key tensors on these prescribed fault planes."*

2. Can you explain why 4 key tensors appears to fit better than 5 key tensors (Figure 8)? Does that have to do with the tensor locations? It seems intuitive that more tensors should offer better fits.

   RESPONSE: You are right; we expect the misfit to go down as we add parameters. This could be attributed to:

   - The stochastic nature of the black-box optimization method (CMA-ES): It was chosen because it is generally very efficient, simple to implement, and thus favorable for prototyping. However, it doesn't guarantee that it will always find the global optimum.
   - The tensor location that is enforced in the chosen parameterization: In order to avoid overlapping tensors, the fault plane was evenly distributed between tensors by cutting the fault plane into vertical zones of even width. The combination of 5 vs 4 and 6 different zones might have been less favorable in terms of fitting the USGS model.

   The following have been added to the description of Figure 8:

   "The misfit increase from four to five key tensors might be explained by a sub-optimal distribution spatial distribution of the key tensor sources or the stochastic nature of the optimization method."

3. I appreciated the author's discussion of potential avenues for determining the kinematics of rupture. Determination of a source time function and/or slip directivity would be a wonderful secondary product for contribution to rapid response.

   RESPONSE: Thank you for your comment. The next step in developing this methodology will involve testing parameterization strategies related to the location and timing of the key tensors. This will be part of a proper data-driven inversion rather than simply reproducing an existing model.

4. Line 36: Should be "ShakeMaps" (capital M)

   RESPONSE: Corrected

5. Figure 9: What is the purpose of the Error scale in the top subplot?

   RESPONSE: As the 0-valued misfit has a slight red-ish tint, to be visible above the whited-out map of the target area, the color-scale has been introduced in all three subplots (even though the first map have all-zeroed out misfit). This figure has been modified as per recommendation of Reviewer C.

6. Figure 11 caption: add commas for clarity: "From top to bottom, upscaled USGS NEIC model, approximated model with n = 9 key tensors, and difference between the two maps."

   RESPONSE: Corrected

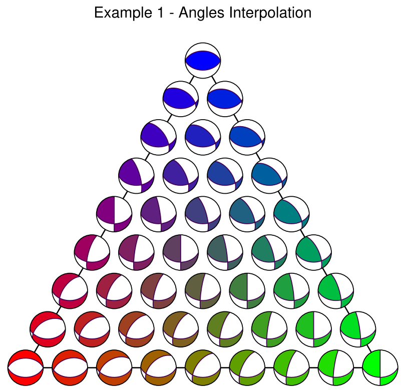

Figure R1: Trilinear tensor interpolation using ranke-angle interpolation.

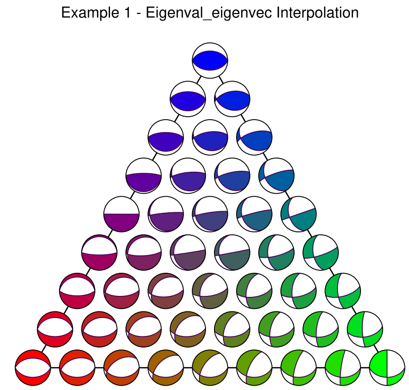

Figure R2: Trilinear tensor interpolation using quaternion on eigenvectors.

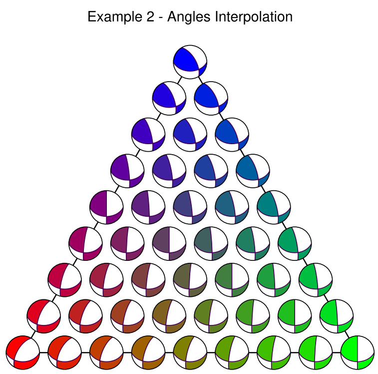

Figure R3: Trilinear tensor interpolation using ranke-angle interpolation.

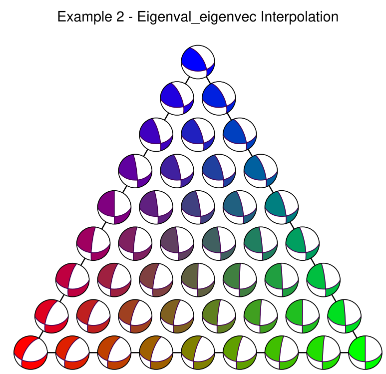

Figure R4: Trilinear tensor interpolation using quaternion on eigenvectors.

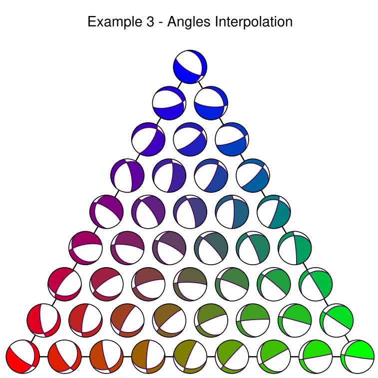

Figure R5: Trilinear tensor interpolation using all angles interpolation.

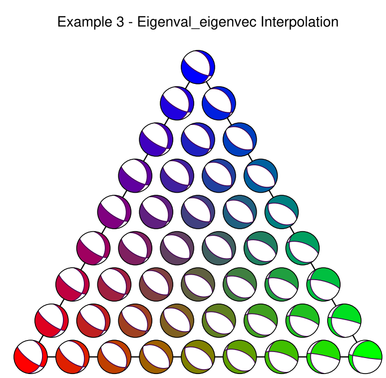

Figure R6: Trilinear tensor interpolation using quaternion on eigenvectors.