REPORT
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A compilation of elastic anisotropy measurements from metamorphic rocks

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Preamble Nikolas Christensen, a pioneer in the study of the elasticity of crust and upper mantle lithologies, passed away on May 19, 2022 (see https://rock.geosociety.org/net/documents/gsa/memorials/v51/Christensen-NI.pdf). Prior to his passing, Nik had been working on a manuscript that described and interpreted his extensive data base of seismic anisotropy measurements. He provided the manuscript materials to one of us (M.G.B.) for comment with the intent of eventual submission. Although the manuscript was not completed, the initial sections, which provide context, describe the methodology, and summarize the measurements, were reasonably self contained. More importantly, the extensive tables of painstakingly taken measurements that form the basis of the work had been prepared in publication-ready form and as spreadsheets. Owing to the profound and invaluable contributions of Nik's previous compilation papers on crustal composition (Christensen and Mooney, 1995) and the isotropic elasticity of common lithologies (Christensen, 1996) to the seismological and geological communities, it is our opinion that the present work should be published for the benefit of future scientific investigations of lithospheric anisotropy. We have elected to submit the manuscript on Nik's behalf as a research note to Seismica. The title has been changed from the original "Metamorphism and crustal seismic anisotropy: A global perspective" to the present one, but aside from this and minor editorial revisions, it is a faithful representation of the original draft. Note that this manuscript is accompanied by an independent commentary by Douglas Schmitt in this issue of *Seismica.*
– Michael G. Bostock, Simon M. Peacock, Matthew S. Tarling. The University of British Columbia.

Abstract An increasing number of seismic investigations have reported convincing evidence for the widespread existence of crustal anisotropy in a variety of tectonic regions. Interpretations of these observations, as well as future seismic studies designed specifically to investigate crustal anisotropy, require detailed knowledge of anisotropic wave propagation in rocks which have undergone deformation and accompanying recrystallization. Of particular importance are the symmetries and magnitudes of P- and S-wave anisotropies and S-wave splitting. A detailed experimental investigation of the anisotropic properties of metamorphic rocks has been carried out to hydrostatic pressures of 1GPa. Each measurement averages the orientations and correct elastic properties of hundreds of thousands of grains, as well as takes into account the important effects of grain shape and grain boundaries on velocities. Common metamorphic rocks, especially those with pelagic protoliths, often have axial symmetries with slow P-wave velocities normal to cleavage, schistosity, and banding. For slates, phyllites, and quartz mica schists, S-wave singularities occur at angles averaging 42° from their symmetry axes, as well as parallel to symmetry axes. Many axial symmetry amphibolites also have slow P velocities and elastic properties similar to crystals with hexagonal symmetry, but unlike metapelitic rocks do not possess off axis S-wave singularities. Rocks with fast axis P-waves and axial symmetry include blueschists, marbles, and dunites. S-wave singularities for these rocks appear to be limited to propagation parallel to symmetry axes. Of importance, maximum S-wave splitting does not always coincide with propagation normal to symmetry axes, and fast vibration directions can be normal as well as parallel to the strike of foliation. Rocks with well-developed foliations and lineations have, as expected, seismic properties similar to those of orthorhombic single crystals. P-wave velocities are fast parallel to lineations originating from foliation crenulations and mineral elongations. Orthorhombic rock S-wave singularities are rare for propagation in mirror planes, but, when present, occur in symmetry planes defined by the maximum and minimum P-wave velocities. Crustal regions most likely to show strong seismic anisotropy include accretionary prisms containing abundant slate and phyllite and crustal regions rich in quartz mica schist and amphibolite.

Key points

- Axial and orthorhombic are common symmetries for many metamorphic rocks, which in turn have elastic properties similar to hexagonal and orthorhombic single crystals.
- Many metamorphic rocks show maximum shear wave splitting for propagation parallel to foliation; however, this is not always the case.
- For rocks with axial symmetry, P-wave propagation parallel to their symmetry axis can either be fast or slow.
- Of the common rock types, slates, phyllites, and quartz-mica schists possess the highest seismic anisotropies.

1 Introduction

A majority of rocks in the Earth's crust have been modified by metamorphic reactions. Tectonic processes responsible for this metamorphism often operate on massive scales affecting large volumes of crustal rock. An important aspect of this regional metamorphism to the seismologist is the formation of rocks that are highly anisotropic to seismic wave propagation.

Seismic anisotropy in metamorphic terranes often originates from preferred mineral crystallographic orientations formed in response to strain (stress). In relatively low-grade metamorphic rocks phyllosilicates undergo physical rotation, sometimes accompanied by recrystallization, to produce mineral alignment and cleavage. In higher grade crustal metamorphism most minerals align by plastic flow and recrystallization with preferred orientations related to schistosity and banding. Of particular importance is the formation of metamorphic terranes with a regularity in the orientations of foliations and lineations. These terranes are prime candidates for investigations of seismic anisotropies of P- and S-waves, which can provide valuable information on regional crustal composition as well as structure.

This paper expands on earlier laboratory studies that have reported rock P- and S-wave velocities in three mutually perpendicular directions. Velocities have been measured in specific directions which allow the rocks to be treated as elastic solids with axial and orthorhombic symmetries, thereby providing stiffness and compliance matrices and complete three-dimensional velocity surfaces.

The following questions are addressed: Which metamorphic rock types are likely to possess P- and S-wave anisotropies of significant magnitudes to be investigated by seismological studies? What are the symmetries of these rocks and how do their seismic velocities vary off axis as well as parallel to symmetry axes? What is the origin and distribution of S-wave singularities in metamorphic rocks (note singularities refer to wavefront directions at which the two S-waves that occur in anisotropic media possess coincident phase velocities)? How does seismic anisotropy change in symmetry and magnitude with progressive metamorphism? Detailed laboratory measurements of velocity anisotropies at elevated hydrostatic pressures for a variety of common metamorphic rocks are, in principle, the best approach to answer these questions. Finally, tectonic regions most likely to be highly anisotropic are briefly discussed.

2 Overview of rock velocity anisotropy

Much of our present knowledge of velocity anisotropy in metamorphic rocks has come from high-pressure laboratory studies, which have investigated the directional dependence of P- and S-wave velocities for a variety of rocks of differing composition and metamorphic grade. Since velocities of below approximately 200 MPa are influenced by grain-boundary cracks, measurements at higher pressures are required to separate the effects of cracks and mineralogy on velocities and their anisotropies. The first systematic studies of velocity anisotropy were made in the 1960s in Francis Birch's laboratory at Harvard University and included P-wave velocity measurements (Birch, 1960, 1961; Christensen, 1965) and S-wave velocity measurements (Simmons, 1964), including S-wave splitting data (Christensen, 1966a). In these early studies, velocities were measured at hydrostatic confining pressures to 1 GPa (equivalent to a depth of approximately 35 km) in three directions from cores taken at right angles to one another from a single sample. Following these studies, several laboratories have provided high-quality ultrasonic velocity measurements, also usually in three directions, for a wide variety of metamorphic rocks, often at pressures high enough to eliminate the lowering of velocities by grain boundary cracks. No attempt has been made here to document fully the large number of papers dealing with velocity anisotropy of metamorphic rocks.

Average P- and S-wave anisotropies, defined as $100\times(V_{max}-V_{min})/V_{avg}$, measured at a hydrostatic pressure of 500 MPa are shown in Fig. 1 for several common crystalline rock types collected from worldwide locations. The majority of the P-wave data (1042 rocks) and S-wave data (761 rocks) used to construct this figure are from the compilations of Christensen and Mooney (1995) and Christensen (1996). In addition, measurements have been added from 45 South Island, New Zealand graywackes, slates, phyllites and schists (Christensen and Okaya, 2007), 6 slates from the central Taiwan slate belt (Christensen, unpublished data), 4 eclogite xenoliths from the Slave Craton, Canada (Kopylova et al., 2004), 22 eclogites from the Dabie ultrahigh pressure metamorphic belt, Eastern China (Zhao et al., 2011), and 6 phyllites and mica schists from Vancouver Island, Canada (Bostock and Christensen, 2012). All measurements, which span a time period of over five decades, were made using the pulse transmission technique described by Christensen (1985). The P-wave and S-wave anisotropies were calculated from velocities

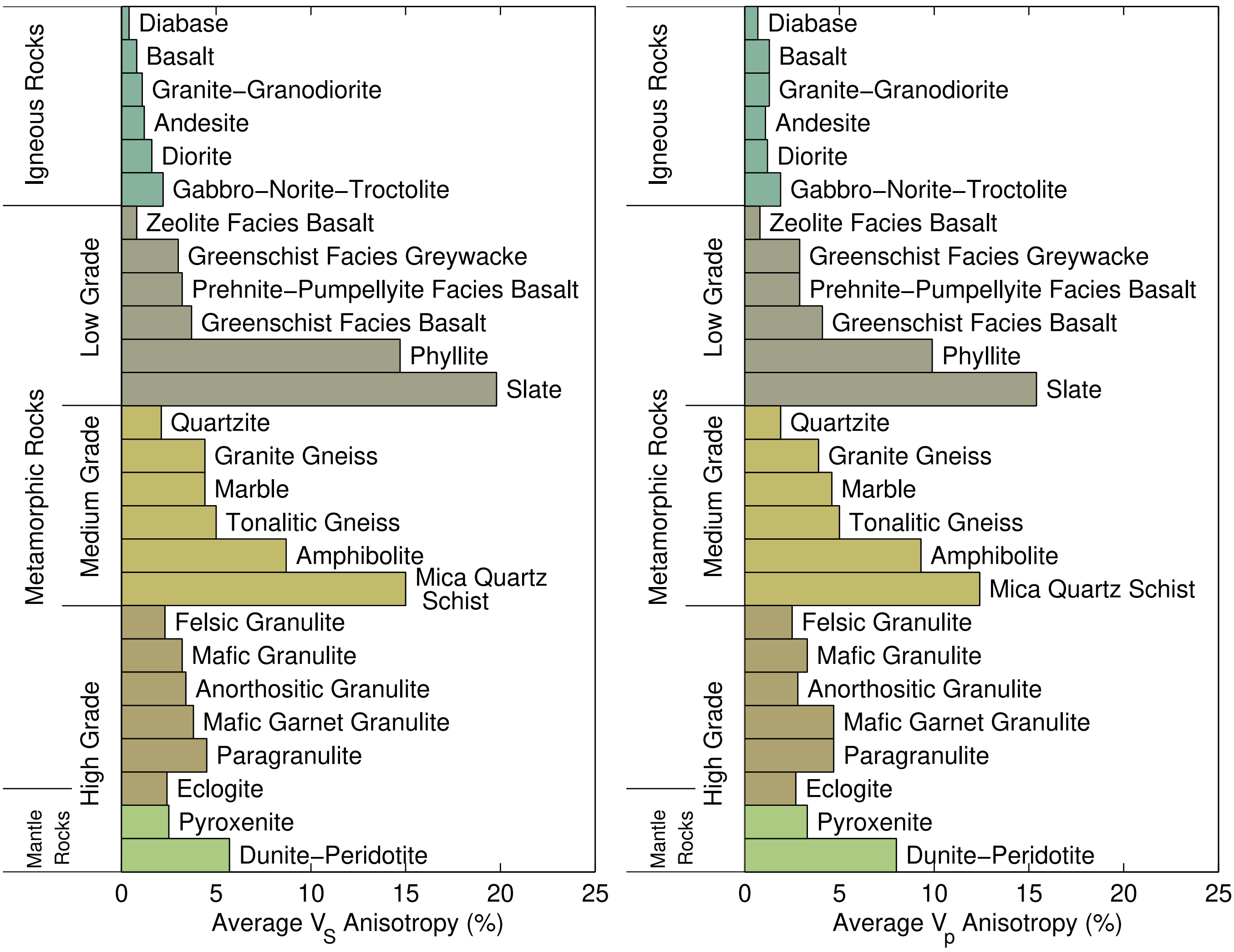


Figure 1 Average rock anisotropies, 100 (V_{max}-V_{min} / V_{avg}), at 500 MPa. Majority of data is from Christensen and Mooney (1995) and Christensen (1996).

measured from 3 orthogonal cores cut from each rock. Two cores were cut with axes parallel to cleavage, foliation, or layering. To obtain information on S-wave splitting in rocks with planar fabrics and/or lineations, two S- velocity pressure runs were made for each core.

Comparisons of these anisotropy measurements provide several important observations. In contrast with most metamorphic rocks, the igneous rocks included in Fig. 1 have relatively low anisotropies, often related to weak mineral alignments. Preferred mineral orientations in igneous rocks are due to alignments of tabular minerals, such as feldspar formed during crystal settling, or fabrics resulting from flow. In addition, some of the calculated anisotropies likely originate from heterogeneity in mineralogy and texture of cores taken from an individual rock.

Most metamorphic rocks have significant anisotropies for both P- and S-waves and rocks with high P-wave anisotropies possess high S-wave anisotropies. Anisotropies are the largest in low-grade phyllites and slates and medium grade amphibolites and quartz mica schists. Anisotropies for these rocks are all higher than the average mantle peridotite. High-grade mafic granulites and eclogites are relatively low in anisotropy. Thus upper and mid-continental crustal regions are more likely to show significant anisotropy than granulite facies mafic lower crust. Regions containing abundant pelitic rocks are likely to be highly anisotropic from shallow depths until reaching granulite facies conditions.

Crustal regions containing abundant basaltic rocks will show a somewhat different pattern of changes in anisotropy with progressive metamorphism, because most lower-grade mafic rocks are not as highly anisotropic as slates and phyllites. This is illustrated in Fig. 1 by following the changes in anisotropy of basalt with increasing metamorphic grade. Anisotropy increases from the zeolite facies through the prehnite-pumpellite facies and greenschist facies to a maximum value at amphibolite facies conditions and then decreases in mafic granulites and eclogites

3 Significance of rock symmetry

One of the most distinctive features of regionally metamorphosed rocks is the preferred orientations of their constituent minerals. Common examples include micas with their 001 planes subparallel to planes of schis-

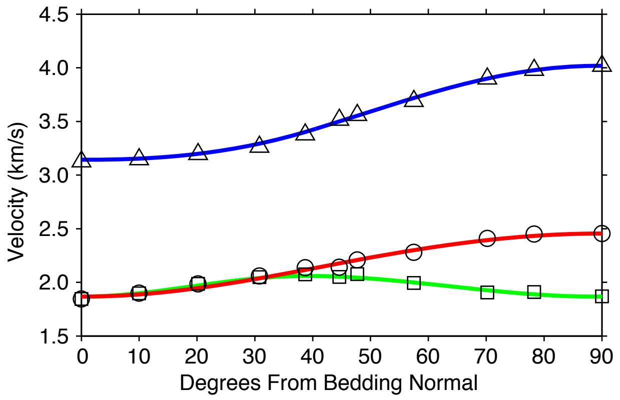


Figure 2 Phase velocity surfaces at 100 MPa for Antrim shale at 100 MPa (Johnston and Christensen, 1995). Measured velocities are shown as symbols, calculated quasi-P velocities are shown as the blue line, calculated S velocities as the red line, and quasi-S velocities as the green line. Note the good agreement between calculated velocity surfaces and measured velocities.

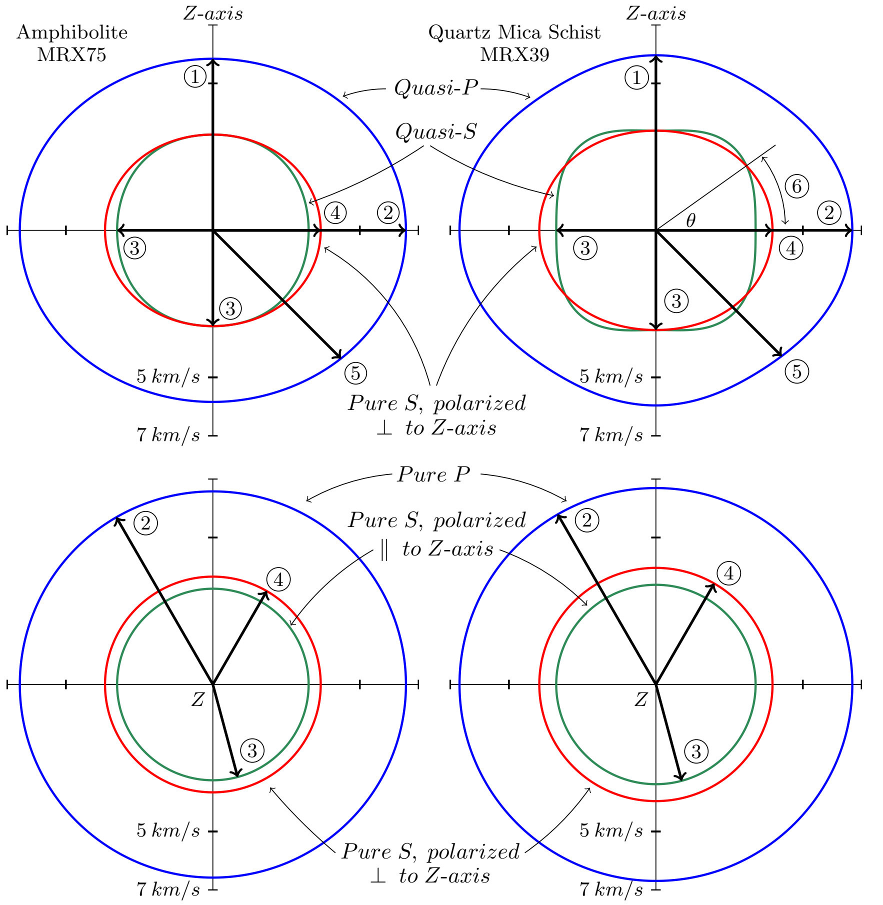
tosity and the preferred orientation of prismatic, tabular, and fibrous minerals producing lineations. These planes and lineations give rise to a rock symmetry, much like the symmetry of a crystal lattice is expressed in terms of point-group symmetry (e.g., Paterson and Weiss, 1961). Rock symmetry is, of course, approximate in that the constituent mineral orientations are statistical, because they are not nearly as ordered as atoms in a crystal lattice. But a close analogy often exists between single crystals and metamorphic rocks since both possess physical properties related to their symmetries (e g., Christensen and Ramananantoandro, 1971; Godfrey et al., 2000). Neumann's principle, a fundamental postulate of crystal physics establishing the relationship between the physical properties of a mineral and its symmetry, states that: **symmetry elements of any physical property of a mineral must include the symmetry elements of the point group of the mineral**. If we apply this principle to mineral aggregates, it follows that the symmetry of a physical property need not be identical to the structural symmetry of the rock, but only needs to include the symmetry elements of the rock (Paterson and Weiss, 1961). Thus the anisotropic elastic properties of a rock cannot have a lower symmetry than the rock, but can be higher. The common symmetries observed in fabrics of deformed rocks are spherical, axial, orthorhombic, monoclinic, and triclinic (Paterson and Weiss, 1961).

The most general form of anisotropic elasticity is that of a triclinic solid with 21 independent elastic stiffness constants, followed by a monoclinic solid with 13 independent elastic stiffness constants (e.g., Musgrave, 1970). These symmetries are common in the most abundant rock forming minerals, including the feldspars, micas, and many varieties of amphibole. Fortunately, the elastic tensors of most common metamorphic rocks can be approximated as having axial symmetry with five independent constants (c_{11}, c_{12}, c_{13} , c_{33}, and c_{44}) or orthorhombic symmetry with nine independent constants (c_{11}, c_{12}, c_{13}, c_{22}, c_{23}, c_{33}, c_{44}, c_{55} , c_{66}).

The symmetry of a rock with an axial fabric is that of a spheroid whose axis is a line of intersection of an infinite number of symmetry planes, in addition to a symmetry plane normal to the axis. A rock is axially symmetric if its appearance is unchanged when rotated around its axis. For axial rocks, calculations of 4 stiffness constants require P- and S-wave velocities measured along symmetry directions parallel and perpendicular to cleavage, schistosity, or banding. The calculation of the fifth stiffness constant requires measuring a quasi-P-wave phase velocity usually at 45° to the planar structure (note the descriptor "quasi-" or "q-" is prepended to P/S-like waves propagating in anisotropic media that are similar to P/S-waves propagating in isotropic media but whose polarizations are not strictly perpendicular/parallel to the wavefront). The elastic properties of axial rocks are similar to hexagonal crystals; however, it is important to note that their axes are not sixfold axes of symmetry.

Rocks with orthorhombic fabrics have the symmetry of a triaxial ellipsoid with three mutually perpendicular symmetry planes and three twofold axes perpendicular to the symmetry planes. For rocks with orthorhombic symmetry, c_{11}, c_{22}, c_{33}, c_{44}, c_{55}, and c_{66} require velocity measurements parallel to the three orthogonal symmetry axes and c_{12}, c_{13} and c_{23} are obtained from quasi-compressional wave velocities measured at 45° to the symmetry axes. To describe three-dimensional wave propagation, phase velocity surfaces can be calculated using the Kelvin Christoffel equations (e.g., Auld, 1990; Musgrave, 1970) and the elastic constants of each sample. These surfaces describe velocities as a function of angle to symmetry planes and axes. Velocity surfaces are calculated for P- and two S-waves. Johnston and Christensen (1995) have shown that these calculated velocity surfaces obtained from measured velocities agree well with independently observed velocity measurements made at several angles to symmetry axes (Fig. 2).

To date only a limited number of metamorphic rocks have had sufficient velocity measurements to completely describe their anisotropy in terms of elastic constants and velocity surfaces, and most of these have axial symmetry. In the following sections elastic constants are presented as a function of pressure for a variety of metamorphic rocks of axial and orthorhombic symmetries believed to be important constituents of the continental crust. These elastic constants are used to produce velocity surfaces which describe variation in P- and S-velocities as a function of angle to the planar and linear structures of the rocks. Multiple samples from worldwide locations have been selected for rocks possessing the highest anisotropies (e.g., slate, phyllite, quartz mica schist, amphibolite, and tonalite gneiss). Velocities have been reported for many of these rocks in three mutually perpendicular directions in previous studies. These measurements and new measurements at 45° to the symmetry axes have provided data for the calculation of complete sets of elastic constants. Measurements have been made to 1 GPa for selected samples in which velocities were previously measured



① $V_p = (c_{33}/\rho)^{1/2}$ ④ $V_s = (c_{66}/\rho)^{1/2}$

② $V_p = (c_{11}/\rho)^{1/2}$ ⑤ $V_{qp} = \dfrac{\left[c_{11} + c_{33} + 2c_{44} + \sqrt{(c_{11} - c_{33})^2 + 4(c_{13} + c_{44})^2}\right]^{1/2}}{(4\rho)^{1/2}}$

③ $V_s = (c_{44}/\rho)^{1/2}$ ⑥ $\theta = \cot^{-1}\sqrt{\dfrac{(c_{13} - c_{44})^2 - (c_{11} - c_{66})(c_{33} - c_{44})}{(c_{44} - c_{66})(c_{11} - c_{66})}}$

Figure 3 Propagation in meridian and foliation planes of hexagonal amphibolite (MRX75) and hexagonal quartz mica schist (MRX39). Note the crossover singularities in the meridian plane of the S-wave velocity surfaces of the quartz mica schist and the absence of S-wave crossovers for the amphibolite.

to 600 MPa and velocities are given in increments of 20 MPa at lower pressures to provide information on the effect of grain-boundary cracks on shallow crustal anisotropy. Details concerning the lithologies, mineralogies, and collection localities of the samples as defined by their labels are provided in Table 1. Table 2 provides velocities and stiffnesses for metamorphic rocks with hexagonal symmetry whereas Table 3 presents these values for seven well-indurated shales originally studied by Johnston and Christensen (1995). Velocities have been added at smaller pressure increments than those reported by Johnston and Christensen (1995). Data from shales are important in that they provide velocity information on the initial stages of metamorphism of pelitic rocks and changes in elastic properties accompanying progressive metamorphism from shale to slate. Many metamorphic petrologists consider diagenesis as transitional with early stages of metamorphism. Finally, Table 4 provides velocities and stiffnesses for metamorphic rocks with orthorhombic symmetry, and Table 5 presents average velocities and stiffness constants for rocks with hexagonal and orthorhombic symmetries along with their sample standard deviations. Tables are available in PDF format alongside this article.

4 Velocity surfaces of rocks with axial symmetry

To a first approximation, a wide variety of metamorphic rocks possess the elastic properties of the crystal classes of the hexagonal system. These rocks have single axes of symmetry. Propagation of elastic waves along the axis is similar to that of isotropic rocks in that there are single P- and S-waves, and the rock exerts no control over the vibration direction of the S-wave. Elastic waves

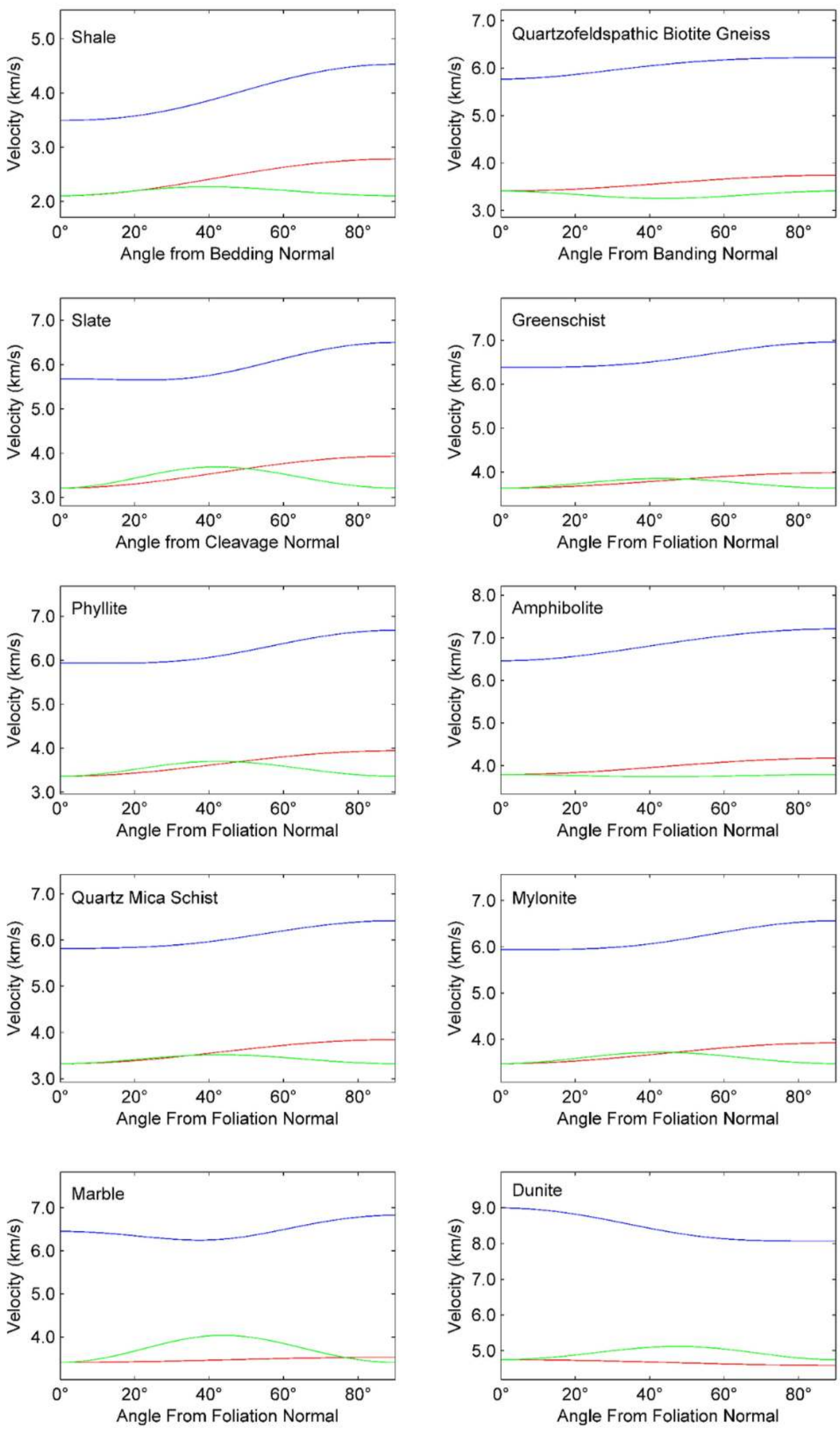


Figure 4 Average quasi-P (blue lines), S (red lines), and quasi-S (green lines) velocities as a function of angle from symmetry axes for common metamorphic rocks. See Table 2 for velocities, densities, stiffness constants, and standard deviations.

propagating normal to the axis are also pure and have circular symmetry about the axis, giving rise to the term transversely isotropic. Elastic wave propagation normal to symmetry axes, however, is different from isotropic rocks in that in addition to a P-wave there are two pure S-waves with perpendicular polarizations. The plane normal to the symmetry axis is a mirror symmetry plane, as are all planes (meridian planes) that contain the symmetry axis. In view of the circular symmetry around the axis, all planes containing the axis have equivalent velocity surfaces. Thus a section illustrating the velocity surfaces in a plane containing the symmetry axis completely describes wave propagation.

4.1 Phase velocity surfaces

Based on previous studies of elastic wave velocities in metamorphic rocks, it appears that a majority possess a

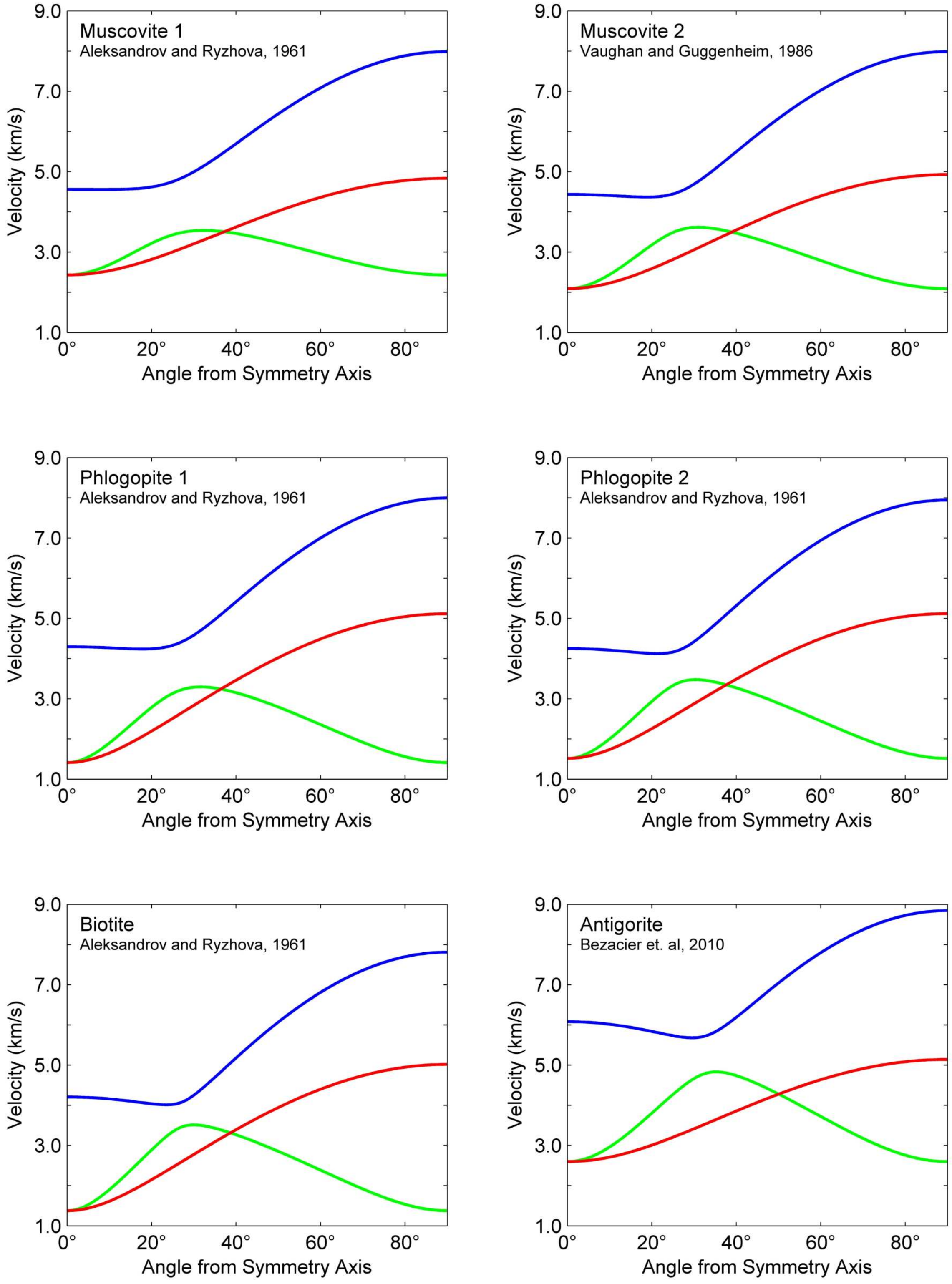


Figure 5 Comparison of propagation in meridian planes of antigorite schists MRX86 and MRX87 with single crystal antigorite (Bezacier et al., 2010) simplified to hexagonal symmetry. Note the similarities in velocity surfaces of the rocks and the single crystal.

symmetry axis normal to slaty cleavage, foliation, or layering with slow P velocities. Two types of phase velocity surfaces, which differ significantly in off-axis wave propagation, are common for these rocks. This is illustrated in Fig. 3, where velocity surfaces for an amphibolite (MRX75) from South Island, New Zealand and a quartz mica schist (MRX39) collected from the Orocopia formation in southern California. The top diagram for each rock shows velocities for propagation in a plane containing the symmetry axis, and the lower diagram shows velocities for propagation in a plane normal to the symmetry axis. Below the velocity surfaces are the

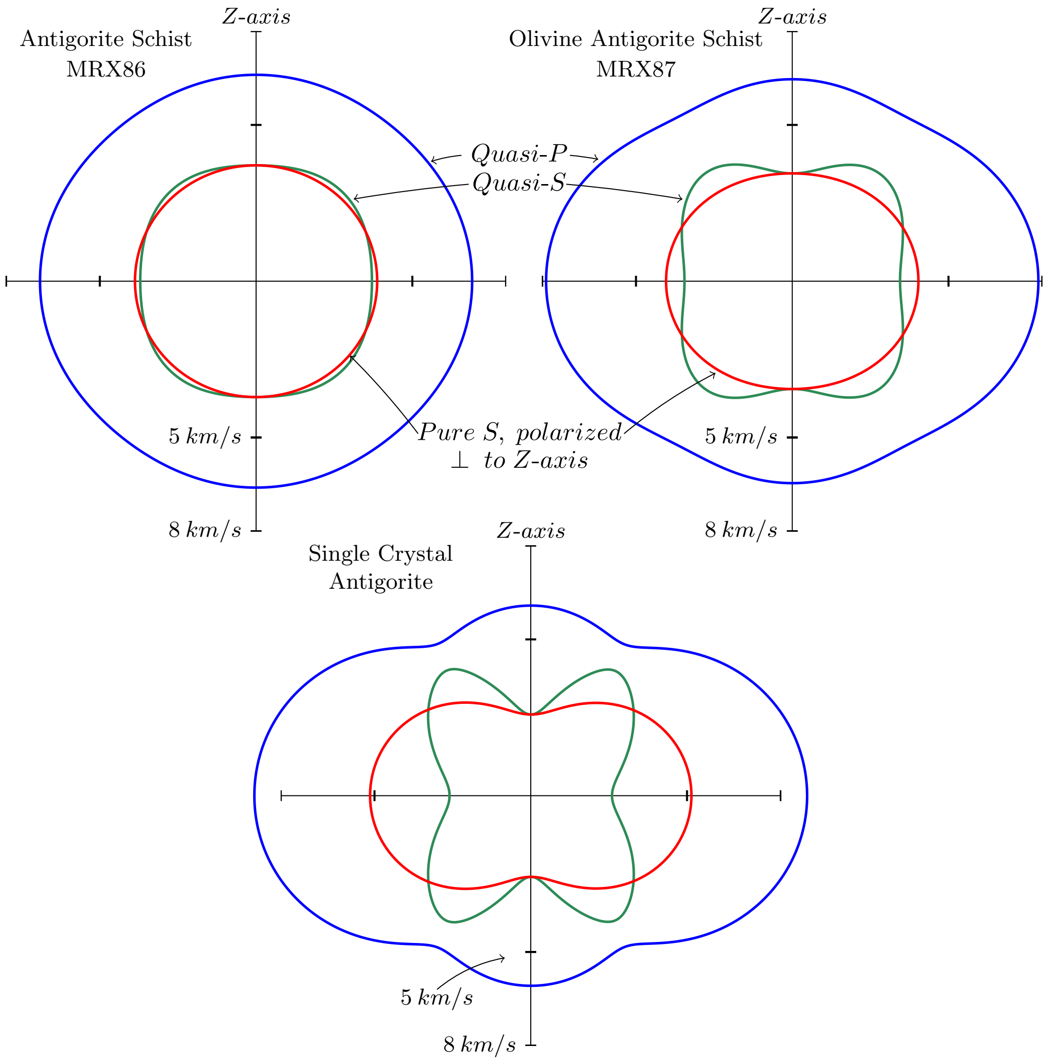


Figure 6 Plots of quasi-P (blue lines), S (red lines), and quasi-S (green lines) velocities vs. angle from lineations for hexagonal symmetry rocks with fast axes (MRX88, MRX90 and MRX99) and velocities vs. angle from foliation normals for MRX91, MRX94, and MRX97.

equations used to calculate the five elastic stiffness constants for these rocks.

For the amphibolite MRX75, splitting is a maximum for propagation parallel to the foliation and gradually decreases as propagation approaches the direction of the symmetry axis. Of importance, the fast S-wave is pure with a vibration direction parallel to the plane normal to the symmetry axis (i.e., the amphibolite foliation). For propagation along the symmetry axis the two S velocity surfaces touch one another tangentially (the “kiss” singularity of Crampin, 1984).

The P phase velocity surfaces for both rocks show smooth decreases in velocity going from propagation in the foliation plane to normal incidence. The velocity surfaces for the pure S waves polarized normal to the symmetry axis also show a smooth increase. S-wave splitting of the quartz mica schist (MRX 39) differs from the amphibolite in that in addition to the singularity for propagation parallel to the symmetry axis there are crossovers of the two velocity surfaces at oblique propagation angles to the foliation, giving rise to circular crossover singularities. Calculation of the angle between the circular singularity propagation direction and the foliation is given by equation (6) in Fig. 3 and is 40° for the quartz mica schist. Note that for propagation in the range of 0° to 40° from the foliation the velocities for the pure S-wave vibrating parallel to the foliation are greater than those for the quasi-S wave. However, for propagation 40° to 90° from the foliation the quasi-S wave has the greater velocity and thus the fast S-wave no longer vibrates parallel to the strike of foliation. It is often assumed that the vibration directions of fast S-waves recorded from field studies map the strikes of foliations or, in some cases, flow directions. However as shown above, this need not be the case for geologic terrains containing rocks with anisotropic elas-

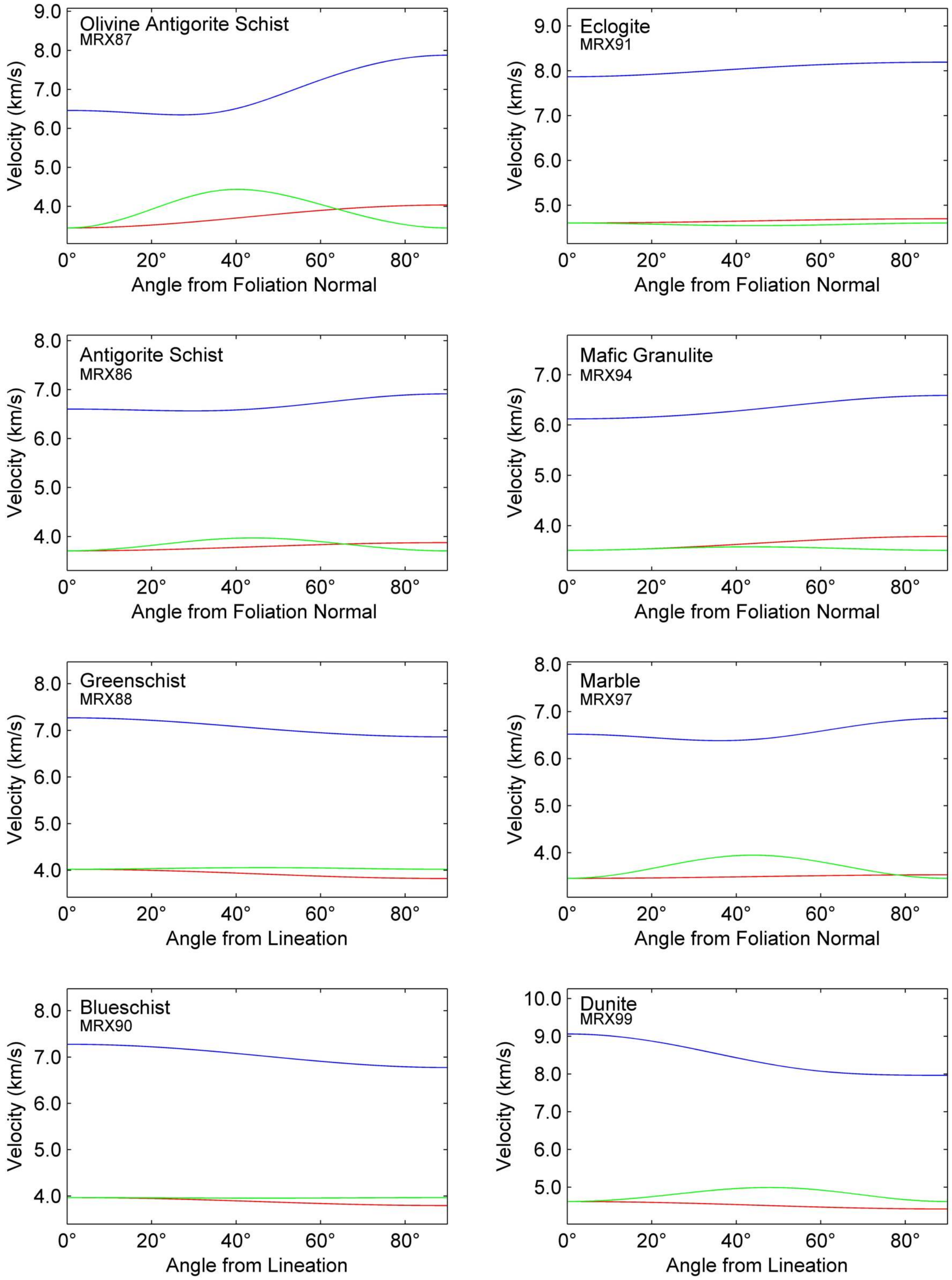


Figure 7 Propagation of quasi-P (blue lines), S (red lines,) and quasi-S (green lines) for single crystal micas and antigorite. All have been simplified to hexagonal symmetry with symmetry axes normal to (001) cleavages. Note crossover singularities of S and quasi-S velocities at angles between 36° and 50° from the symmetry axes.

tic properties similar to MRX75 (i.e. rocks with similar crossover singularities, e.g., Bostock and Christensen, 2012). In the next section we will investigate in detail this behavior of S-wave splitting for a wide variety of common metamorphic rocks

4.2 Average phase velocity surfaces

Because there are often significant differences in the mineralogies of correctly classified rocks, physical property measurements often show much variability for a given rock type. This is particularly true for metamorphic rocks whose classifications are often based on metamorphic grade (e.g., granulite) or texture (e.g., schist) as well as mineralogy. Thus when discussing properties of metamorphic rocks it is particularly important to keep in mind the relevant mineralogy.

Elastic constant data for multiple samples are given for many of the metamorphic rocks with the highest anisotropies in three directions as shown in Fig. 4. Average elastic constants and velocities as a function of pressure along with standard deviations are given in Table 5 for slate, phyllite quartz mica schist, quartzofeldspathic biotite (tonalite) gneiss, greenschist, and amphibolite. In addition, average data are reported for shale and mylonite. Average calculated velocities as a function of angle to planar elements are shown in Fig. 4. Due to their axial symmetry, only 90° of data are required to completely define velocities in a plane containing the symmetry axis. All velocity curves, with the exception of that for shale, are shown at midcrustal pressures of 600 MPa.

Metapelitic rocks, in general, tend to show similar S-wave velocity crossover singularities discussed earlier for MRX-39. The average shale has a very weak crossover at a propagation angle of 23° to the bedding normal. Similar crossovers are better defined for the average slate and phyllite, with crossovers at 50° and 48°. The quartz mica schists show the most variability of the lithologies in Fig. 4: the average quartz mica schist has a weak crossover at 35° with practically no shear wave splitting for propagation directions from 35° to 0°. Several individual schist samples have no crossovers.

An S-wave velocity crossover singularity is present for the average mafic greenschist at 49.6° to the foliation normal. The average mylonite is quite similar to the average phyllite, with the S-wave velocity crossover at 46° and velocities almost identical to the average phyllite. This is not surprising because of the similarities in their mineralogies and textures. Many of these rocks are from the Brevard zone located in South Carolina and in hand sample can easily be mistaken for phyllites. The Brevard zone mylonites have crystalloblastic textures and in thin section it is clear that they have been highly sheared and recrystallized (Reed and Bryant, 1964). Chemical analyses of the samples reveals that they were formed from two major protoliths, granite and shale (Christensen and Szymanski, 1988). The rocks which have formed from granite or granite gneiss have an average shear velocity crossover of 44°, whereas those with chemistries similar to shale have an average crossover of 55°

4.3 Origin of circular S-wave crossover singularities in micaceous rocks

The velocity surfaces derived from the Christoffel equation for the amphibolite and quartz mica schist shown in Fig. 4 differ significantly in the behavior of their S-waves. Much of this is due to the shape of the quasi-S-wave phase velocity surface. In particular, as discussed above, the circular crossover singularities observed for the quartz mica schist have important implications in the interpretations of S-wave splitting observations. Both rocks are of similar metamorphic grade and have well-developed foliations defined by planar preferred orientation of grain boundaries and to a lesser extent by lithologic layering. Neither rock shows a significant lineation in hand sample and thin section.

As discussed in the last section, the velocity surfaces derived from the Christoffel equation for the amphibolite (MRX75) and the quartz mica schist (MRX39) differ significantly in the behavior of the S-waves. Much of this is due to the shape of the quasi-S wave velocity surface and its intersections with the pure S-wave velocity surface (Fig. 3). These circular crossover singularities in the quartz mica schist, or the lack of singularities as observed for the amphibolite, have important implications in the interpretation of S-wave splitting observations. Both rocks are of similar metamorphic grade and have well-developed foliations, defined by planar preferred orientations of grain boundaries and to a lesser extent lithologic layering. Neither rock shows a significant lineation in hand sample or thin section. Thus it appears that the crossover singularities originate, at least in part, from mineralogy.

The origin of the crossover singularities appears to be related to the inherent elastic properties of their mineral constituents. Phyllosilicates are common constituents of most low and medium grade metamorphic rocks. Elastic constants for some varieties, primarily micas, have been reported beginning with the pioneering work of Aleksandrov and Rhyzhova (1961a,b). Their study reported elastic constants for single crystals of muscovite, phlogopite, and biotite, based on ultrasonic pulse transmission velocity measurements. Although most micas are monoclinic, they were treated as hexagonal with their axes of symmetry normal to their well-developed cleavages. Since these minerals are pseudohexagonal the errors in this simplification are only a few percent. Using Brillouin scattering Vaughan and Guggenheim (1986) measured acoustic velocities in single crystal muscovite and reported the 13 elastic moduli characteristic of monoclinic crystals. They also concluded that the departures of the monoclinic elastic constants from hexagonal are minor. More recently, using Brillouin spectroscopy, Bezacier et al. (2010) measured elastic constants of the phyllosilicate antigorite, a major serpentine mineral stable at high temperatures. Like muscovite, antigorite is monoclinic with elastic properties very close to that of hexagonal minerals.

In Fig. 5 velocities are shown as a function of propagation angle relative to the hexagonal symmetry axis (perpendicular the cleavage) for the five micas and antigorite. Note that for all samples the P-velocity is a maximum for propagation normal to the symmetry axis and P-velocities are low for propagation parallel the symmetry axis and remain slow or decrease slightly for propagation directions up to approximately 25°- 30° to the symmetry axes. Also, quasi-P-wave velocity does not change significantly as the propagation direction varies from normal to approximately 80° to the symmetry axis.

All samples show maximum S-wave splitting when the propagation direction parallels the mineral cleavage. For this propagation direction the velocity surface of the fast S-wave (vibrating in the plane of the cleavage) is ellipsoidal in shape and shows a smooth decrease in velocity as the propagation direction approaches the symmetry axis. A dramatic peak in the quasi-S wave velocity is observed for each single crystal at propagations approximately 30° from the symmetry axes. This produces crossover singularities at angles from the symmetry axis of 36.4° to 38.9° for the micas and 50.0° for the single crystal antigorite. Thus the velocity surfaces of the mica single crystals have striking similarities to that of the quartz mica schist

MRX39, indicating that the crossover singularity observed in the rock originates from its mica content. Later it will be shown that a similar crossover singularity is present in antigorite schists which originates from the elastic properties of single crystal antigorite.

4.4 Velocity surfaces for some additional rock types with axial symmetry

The compilation provides velocity surfaces for several rock types from a variety of tectonic environments. This includes antigorite schists believed to be important constituents of mantle wedges, high-pressure rocks from subduction zones, lower crustal granulite, marble, and mantle dunite. Velocity surfaces for several of these rocks with approximate hexagonal symmetry are shown in Fig. 6. Some differ significantly from the velocity patterns of the rocks previously shown in Fig. 3.

Although antigorite, one of the three principal varieties of serpentine, like mica is a sheet silicate, there are major differences in the details of the structures which are related primarily to their compositional differences and periodic inversion of antigorite's layers, creating a corrugated sheet structure. This results in quite different elastic properties of antigorite-bearing rocks when compared with serpentinites consisting of lizardite and chrysotile (e.g., Birch, 1960; Christensen, 2004, 1966b). All four antigorite schists included in this study (MRX84, 85, 86 and 87, see Table 1) are hexagonal with slow axes of symmetry and S-crossover singularities. Velocity curves for two of the samples (MRX87 and 86) are shown in Fig. 6. Note that the crossovers occur at higher angles to the foliation normal then those of the micaceous rocks (Fig. 5). This is in agreement with the plots for the single crystal shown in Fig. 5. The crossover for single crystal antigorite is at 50° to the symmetry axis, whereas the five micas in Fig. 5 have crossovers ranging from 36.4° to 38° to their symmetry axes.

In addition to the antigorite schists from the North Cascades, Washington, velocity curves are shown in Fig. 7 for 3 rocks originating from subduction zone metamorphism. MRX91 is an eclogite from the Dabie ultra high-pressure metamorphic belt in Eastern China and MRX88 and MRX90 are a greenschist and blueschist from the northern sequence of the Chugach terrain of southeastern Alaska (schists of Liberty Creek). Anisotropies of the Dabie eclogites are quite variable, often originating from mineralogical layering and retrograde metamorphism, in addition to metamorphic foliation (e.g., Zhao et al., 2011). The rock included in this study has hexagonal symmetry originating from layering and foliation and a lack of lineation. Note the weak splitting for propagation directions 28° to 90° from the foliation normal.

The blueschist and greenschist rocks are from an exposed subduction zone complex located in Alaska along the boundary of the Wrangellia and Chugach terrains. Metamorphism occurred in the early Jurassic. Maximum computed pressure and temperature for the blueschist are 1.6 GPa and 250° to 280°C, respectively, indicating a subduction depth of 50 to 55 km (López-Carmona et al., 2011). Both rocks are multiply deformed, have strong lineations, and unlike most rocks with hexagonal symmetry have fast axes (Brocher and Christensen, 1990). The lineations originate primarily from strong preferred orientations of amphibole *c* crystallographic axes in the blueschist and trains of elongated epidote crystals in the greenschist. P-wave velocities decrease as the propagation angle from the lineations increases. Also, maximum splitting occurs at propagation normal to the lineations.

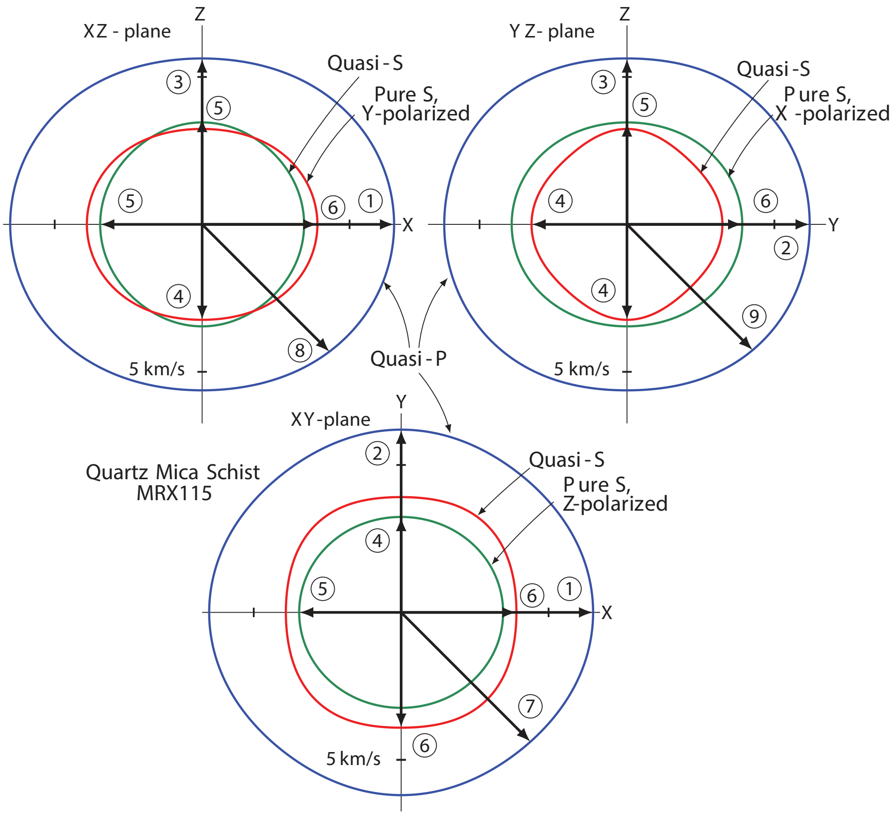
The marble MRX97 has a well-developed foliation, normal to a slow axis of symmetry. The dunite MRX 100 from Cypress Island, Washington, on the other hand, has a fast axis of symmetry parallel to a strong lineation defined by plastically deformed olivine grains elongated parallel to their *a* crystallographic axes. Fabric studies by Raleigh (1965) have shown that olivine in the Cypress Island ultramafic rocks have strong olivine *a* axis maxima, and *b* and *c* axes girdles. Since the fastest P velocity in olivine is the *a* crystallographic direction, this fabric produces hexagonal symmetry with a fast axis. Both the monomineralic marble and dunite in Fig. 7 have maximum shear wave splitting at 40 to 50° to the foliation normal in the marble and the dunite lineation

5 Velocity surfaces of rocks with orthorhombic symmetry

Sixteen rocks (8 schists, 5 amphibolites, 1 slate,1 phyllite, and 1 dunite) with orthorhombic symmetries are included in this study. All have well-developed foliations and lineations. The amphibolite lineations are defined by preferred orientations of prismatic or tabular amphibole crystals lying within foliations. This differs from the hexagonal amphibolites which have nearly random orientations of elongated amphibole *c* axes in their foliation planes. The lineations of the quartz mica schists and lower grade metapelites are defined by small amplitude penetrative crenulations of their foliations. The lineation in the dunite (MRX99, Table 1) originates from elongation of olivine grains parallel to their *a* crystallographic axes.

Solids with orthorhombic symmetry have three mutually orthogonal mirror planes, the intersections of which are twofold axes of symmetry. Like propagation in meridan planes of hexagonal materials, propagation in orthorhombic symmetry planes includes a pure shear wave polarized normal to the plane, a quasi-S wave and a quasi-P-wave. All three modes are pure for propagation along the twofold symmetry axes, with the S-waves vibrating in the symmetry planes whose intersections defines the symmetry axes.

To obtain the necessary velocity data for calculation of nine elastic constants, cores were taken in six direc-



① $V_p = (c_{11}/\rho)^{1/2}$ ② $V_p = (c_{22}/\rho)^{1/2}$ ③ $V_p = (c_{33}/\rho)^{1/2}$

④ $V_s = (c_{44}/\rho)^{1/2}$ ⑤ $V_s = (c_{55}/\rho)^{1/2}$ ⑥ $V_s = (c_{66}/\rho)^{1/2}$

⑦ $$V_{qp} = \frac{\left[c_{11} + c_{22} + 2c_{66} + \sqrt{(c_{11} - c_{22})^2 + 4(c_{12} + c_{66})^2}\right]^{1/2}}{(4\rho)^{1/2}}$$

⑧ $$V_{qp} = \frac{\left[c_{11} + c_{33} + 2c_{55} + \sqrt{(c_{11} - c_{33})^2 + 4(c_{13} + c_{55})^2}\right]^{1/2}}{(4\rho)^{1/2}}$$

⑨ $$V_{qp} = \frac{\left[c_{22} + c_{33} + 2c_{44} + \sqrt{(c_{22} - c_{33})^2 + 4(c_{23} + c_{44})^2}\right]^{1/2}}{(4\rho)^{1/2}}$$

Figure 8 Phase velocity surfaces in the three symmetry planes of orthorhombic quartz mica schist MRX115. The XY plane is the foliation, Z is normal to the foliation, and X parallels the lineation.

tions from each sample. Three of the cores were cut parallel to the twofold symmetry axes : (1) a direction in the foliation (the XY plane) parallel to the lineation (direction X), (2) a second in the foliation and normal to the lineation (direction Y) and (3) normal to the foliation (direction Z). The remaining three cores were taken in the symmetry planes at 45° to the symmetry axes.

The relations between wave velocity, the directions of propagation and displacement, and the stiffness constants for a rock with orthorhombic symmetry are shown in Fig. 8 along with phase velocity surfaces calculated from the stiffness constants at 600 MPa for propagation in the three symmetry planes of a quartz mica schist from Torrington, Connecticut (MRX115, Table 1). For this rock, as with all of the orthorhombic samples, P-wave velocities normal to the foliation are slow. P-wave velocities vary within the foliations and are fast parallel to the lineations. There is S-wave splitting for propagation parallel to the three twofold symmetry axes and for all propagation directions within the YZ and XY foliation planes. Another feature, common to all of the orthorhombic samples including the slate MRX108, phyllite MRX109, and dunite MRX123 (all Table 1) is the presence of shear wave singularities in the XY plane. The singularities are at 30° from the foliation normal for MRX115.

Of the eight quartz mica schist samples, six show velocity surfaces similar to those of MRX115 with crossover singularities limited to the symmetry plane containing the lineation and foliation normal (XZ plane). The remaining two samples, MRX110 and MRX113, have crossovers in the YZ plane in addition to the XZ plane. These two samples have symmetries close to hexagonal, in that propagation along Z shows minimum S wave splitting and P velocities in the foliation plane differ by only a few percent. Velocities in the two planes containing the Z axis are not identical, which they would be for rocks with hexagonal symmetry.

The velocity surfaces of the orthorhombic amphibolites are remarkably similar to one another and to the orthorhombic quartz mica schists. Average values for the two lithologies can be found in Table 5. The maximum anisotropy for P-waves is in the ZX plane for both rock types. This differs from slow axis hexagonal schists and amphibolites for which the maximum anisotropy occurs for propagation within the foliation plane. Also for both rock types maximum S-wave splitting occurs within the foliation plane at 40° to 50° to their lineations. Both rocks show a characteristic orthorhombic S-wave crossover singularity in their ZX planes. At 600 MPa the crossover is at 50° and 57° from their Z-axis for the average schist and amphibolite. For most rocks there is little change in position of the crossovers at pressures between 100 MPa and 1 GPa. However, at lower pressures there is a tendency for the crossover to move towards the Z-axis with increasing pressure.

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Data Availability

Tables 1-5 are provided in PDF format alongside this article. Associated supplemental data can be accessed as spreadsheets in a Borealis data repository (Christensen, 2026).

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- **Table 1.** Sample localities, lithology and mineralogy. See also Supplement spreadsheet "Sup.1 Locations, RockTypes, Mineralogy Table.xlsx"
- **Table 2.** Velocities and stiffness constants for rocks with hexagonal symmetry. See also Supplement spreadsheet "Sup.2 Hexagonal and Orthorhombic Master Table.xlsx".
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