

Recalibration of the local magnitude scale of the Belgian seismic network: Improved models of geometrical spreading and path duration

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Abstract The current local magnitude (M_L) scale for local and regional earthquakes recorded by the Belgian seismic network was developed in 1985 using Lg-wave amplitudes measured on paper seismograms from vertical seismographs, and has been applied to records from digital seismometers installed progressively since then. However, the seismic attenuation in this scale does not properly reflect the relative contribution of geometrical spreading and anelastic attenuation of crustal S-waves at distances < 150 km. Furthermore, the scale is not well constrained at distances $< \sim 20$ km, encumbering magnitude evaluation of shallow small earthquakes (e.g., induced seismicity) recorded only by nearby stations. We recalibrate the Belgian M_L scale as a function of hypocentral distance, paying particular attention to short distances, while keeping good agreement with the original scale at larger distances. The recalibrated scale features a segmented power-law geometrical spreading model, shows improved residuals, and yields lower magnitudes for earthquakes with $M_L < \sim 1.5$. We also performed a calibration using horizontal components, but residuals are significantly higher, hampering adoption. Nevertheless, these results have led to improved models of geometrical spreading and path duration for horizontal S-waves, which are important components in many seismological applications. Finally, we also derive an empirical relation between M_L and moment magnitude.

Non-technical summary The magnitude scale currently used to determine the size of earthquakes in Belgium has two main drawbacks: 1) it is based on horizontal distance, and hence does not consider earthquake depth, and 2) it is poorly constrained at short distances. As a result, it is difficult to apply to shallow, small earthquakes, which are typically only recorded by nearby seismic stations. In this study, we recalibrate the Belgian local magnitude scale as a function of hypocentral distance using digital seismic data recorded since 1985. In doing so, we pay particular attention to short distances, while ensuring good agreement with the original scale at larger distances. The recalibrated scale is valid for shorter distances ($> \sim 6$ km), shows a better fit with the data, and gives lower magnitudes for small earthquakes. The original scale used only the vertical component, but we also perform a calibration using the horizontal components. However, as the uncertainties are larger, we prefer to continue using the vertical component. On the other hand, these results give us better insight in the attenuation of horizontal S-waves, which is important for many seismological applications. Finally, we also establish a relation between the Belgian local magnitude scale and standard moment magnitude.

1 Origin and issues of the Belgian local magnitude scale

The local magnitude (M_L) scale that is currently used to determine the size of local earthquakes recorded by the Belgian seismic network, hereafter referred to as the “Belgian M_L scale”, was developed by Camelbeeck (1985, 1993). The scale is based on the attenuation of the vertical displacement amplitude of the 1 Hz Lg wave, which was also used to develop a magnitude scale for shallow (1.5 – 15 km depth), small to moderate earthquakes in eastern North America (Nuttli, 1973; Her-

rmann and Kijko, 1983b; Herrmann and Nuttli, 1983), where the scale was denoted as M_N (with N referring to Nuttli) or m_{bLg} (magnitude based on 1 Hz Lg wave calibrated with respect to teleseismic P-wave m_b).

Lg waves are thought of as multiply reflected shear waves (Atkinson and Mereu, 1992; Sargeant and Ottemöller, 2009) or as a superposition of higher-mode surface waves (Herrmann and Kijko, 1983a) propagating in the crustal wave guide with a group velocity of ~ 3.5 km/s. They have frequencies between 0.2 and 5 Hz, and usually constitute the largest-amplitude arrival on short-period seismograms at regional distances between 150–200 km up to >1000 km, which makes them very suitable to define a magnitude scale (Herrmann

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and Kijko, 1983a; Herrmann and Nuttli, 1983). Nuttli (1973) modelled the attenuation of the Lg vertical displacement amplitude A (μm) as a function of epicentral distance R_e (km) as:

$$A(R_e) = K_1 R_e^{-\frac{5}{6}} \exp(-\gamma R_e) \quad (1)$$

In this equation, which is valid up to a distance of 1000 km, K_1 is a constant corresponding to the amplitude at a reference distance (usually 1 km) and the two R_e terms represent geometrical spreading and anelastic attenuation of the 1 Hz Lg wave, respectively, with γ corresponding to the spatial anelastic attenuation factor (units of km^{-1}). It can be noted that γ is related to equivalent regional attenuation parameters like the quality factor Q and the gradient of the high-frequency attenuation parameter κ (Anderson and Hough, 1984; Ktenidou et al., 2015):

$$\begin{aligned} \gamma &= \frac{\pi f}{Q\beta} \\ &= \pi f \frac{\delta\kappa}{\delta R} \end{aligned} \quad (2)$$

where f is the frequency (1 Hz in this case) and β the group velocity (km/s) of the Lg wave.

Local magnitude has been defined (Richter, 1935) as:

$$M_L = \log_{10} A(R) - \log_{10} A_0(R) \quad (3)$$

where $A(R)$ denotes maximum displacement amplitude observed at (usually epicentral) distance R , and $A_0(R)$ corresponds to the maximum displacement amplitude of a magnitude zero earthquake at the same distance.

Combining Equations 1 and 3, Nuttli (1973) derived a formulation for a magnitude scale based on vertical Lg displacement amplitudes for eastern North America, which can be written in a general form (Herrmann and Nuttli, 1983) as:

$$\begin{aligned} m_{bLg} &= \log_{10} A(R_e) \\ &- \left(\log_{10} K_1 - \frac{5}{6} \log_{10} R_e - \gamma R_e \log_{10} e \right) \end{aligned} \quad (4)$$

The m_{bLg} scale was selected for application in Belgium because it had been demonstrated (Herrmann and Kijko, 1983a) that m_{bLg} evaluation is stable for a wide range of instruments, and because it is based on the vertical component, which was a big advantage considering the lack of 3-component instruments at the time. For application in other regions than eastern North America, it is necessary to determine γ and calibrate K_1 (Herrmann and Nuttli, 1983). Camelbeek (1985) determined γ from two datasets of single-station paper records of earthquakes in central and southern Europe up to ~800 km distance: (1) 39 events recorded on the vertical Benioff seismometer at station Dourbes (DOU) during the year 1980, and (2) 13 events recorded by the Sprengnether series D-H seismometer at Warmfontaine (WRM) during the period 1965-1971. The magnitudes for the former were taken from the Laboratoire de Détection et de Géophysique (LDG) network, while

those for the latter were provided by the Bureau Central de l'Association Internationale de Sismologie (BCIS, now BCSF). The γ values obtained for the two datasets were 0.0026 km^{-1} and 0.0023 km^{-1} , respectively. These values are comparable, but the former one was adopted, as it was based on more data and considered to be more robust. The corresponding K_1 value was chosen such that the reference magnitude ($m_{bLg-R_{ref}} = 3.0$) yields a displacement of $2.42 \mu\text{m}$ at $R_e = 10 \text{ km}$, i.e.

$$\log_{10} K_1 = \frac{5}{6} + 10\gamma \log_{10} e + \log_{10} 2.42 - m_{bLg-R_{ref}} \quad (5)$$

Following the rollout of a digital seismic network in Belgium from 1985 onward, Camelbeek (1993) performed a joint inversion to determine γ , M_L , and station residuals simultaneously for all stations and earthquakes using digital records collected since that time. It is important to note that all distances concerned were less than 200 km, implying that most measured amplitudes do not strictly correspond to the Lg wave, but rather to the S wave (Sn and SmS phases), which transitions into the Lg wave at $R_e > \sim 150 \text{ km}$. Camelbeek (1993) obtained an attenuation curve similar to the one for m_{bLg} based on analogue data, with a maximum magnitude difference of 0.1, and remaining stable with distance. Thus, the coherence with the m_{bLg} scale was clearly established, and henceforth the latter has been applied systematically to all instrumental earthquakes recorded by the Belgian seismic network since 1985 and considered equivalent to M_L .

In contrast to the Belgian M_L scale, the definitions of the M_L scales currently used by neighbouring networks in The Netherlands (Royal Netherlands Meteorological Institute, KNMI) (Dost et al., 2004; Kruiver et al., 2021) and Germany (Bensberg network of the University of Cologne) (Hinzen et al., 2020; Reamer and Hinzen, 2004) are based on horizontal S-wave displacements on a simulated Wood-Anderson seismograph. Nevertheless, the M_L values determined by the Belgian network show a good overall agreement with those reported by these neighbouring agencies, as shown for common earthquakes in the border region between Belgium, Germany and The Netherlands since 2000 (Figure 1). The variability of M_L differences does increase with lower magnitude, but this is at least in part due to the larger amount of data.

In the past two decades, the Belgian seismic network has been further improved, resulting in a lower detection threshold. This has brought to light two major drawbacks of the current M_L scale:

- The distance metric of the scale is epicentral distance, rather than hypocentral distance (R_h). This was a logical choice for a scale based on the Lg wave, which only emerges at distances beyond 150 – 200 km, but much less when it is applied to the S wave at distances that are mostly less than 200 km. As a result, identical earthquakes at different depths would be assigned different magnitudes, as their recorded amplitudes would be different. On the other hand, earthquakes generating similar amplitudes at similar distances would be as-

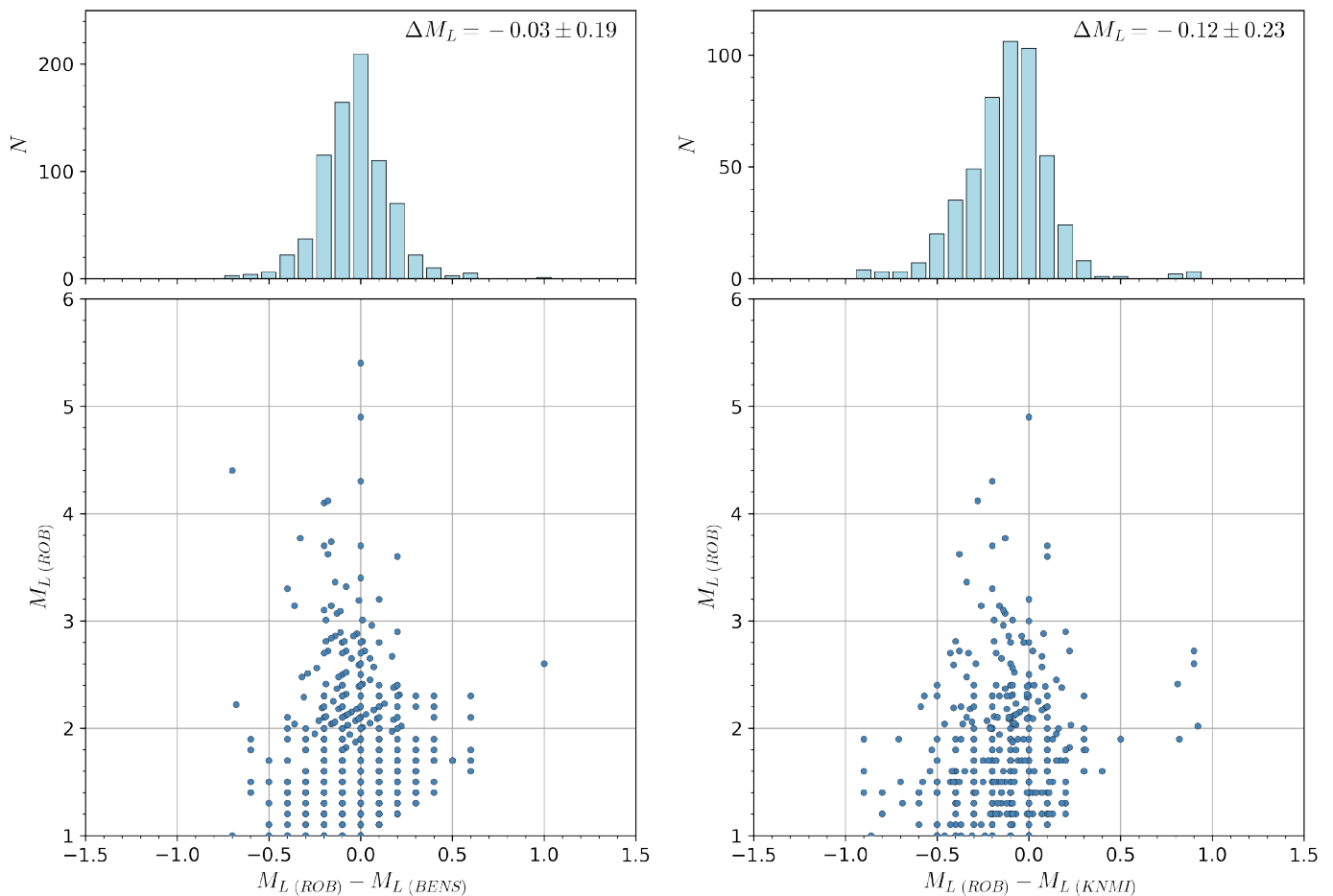


Figure 1 Differences between M_L values determined by the Belgian network (labeled “ROB”) and by neighbouring networks for common events with $M_L \geq 1.0$ since 2000. Left: compared to Bensberg network of the University of Cologne, Germany. Right: compared to KNMI network, The Netherlands. Bottom panels show M_L differences versus M_L , top panels show histograms.

signed the same magnitude, regardless of their focal depth.

- Due to the lack of short-distance amplitudes when the scale was established, ground-motion attenuation is poorly characterized for distances shorter than ~ 20 km, which makes it difficult to determine M_L for smaller earthquakes that are only recorded by nearby stations. This is an issue for monitoring induced seismicity, which started only a decade ago in Belgium, as operators need to implement a traffic light system based on small earthquakes that are generally only detected by local networks.

2 Objectives and data

In this study, we aim to adjust the Belgian M_L scale for short distances (few tens of km), while ensuring a good agreement with the original scale at larger distances, where the latter was defined. Considering the good agreement (within ± 0.1 magnitude units) of the original M_L scale with the scales of neighbouring agencies, and for reasons of continuity, we do not seek to establish an entirely new scale, but rather to recalibrate the original scale as a function of hypocentral distance. In addition, we also want to test whether it is possible to calibrate the M_L scale using horizontal components, similar to cur-

rent practice of neighbouring networks and considering that most instruments in the Belgian network have three components now.

As station amplitudes have not been consistently stored for older events in the database containing all routine seismic data of the Royal Observatory of Belgium (referred to as “ROB catalog” henceforth), we will use data from the BELSHAKE database (Vanneste et al., 2026). This is a ground-motion database containing all available digital records and associated metadata of ~ 340 natural and induced earthquakes with $M_L \geq 2$ in and around Belgium since 1985 (Figure 2), recorded with short-period, broadband, and accelerometric instruments operated by ROB. All waveform data have been visually inspected to correct or flag problems such as low signal-to-noise ratio, timing problems, disturbances, clipping, spikes, baseline distortion, interruptions, missing or incomplete phase windows, anomalous component amplitudes, and overlap with foreshocks or aftershocks. Instrument response information was readily available for waveform data recorded since ~ 2005 or later, depending on the seismic station. However, for older waveform data, this information had to be reconstructed from various sources of information, as described in more detail in Vanneste et al. (2026).

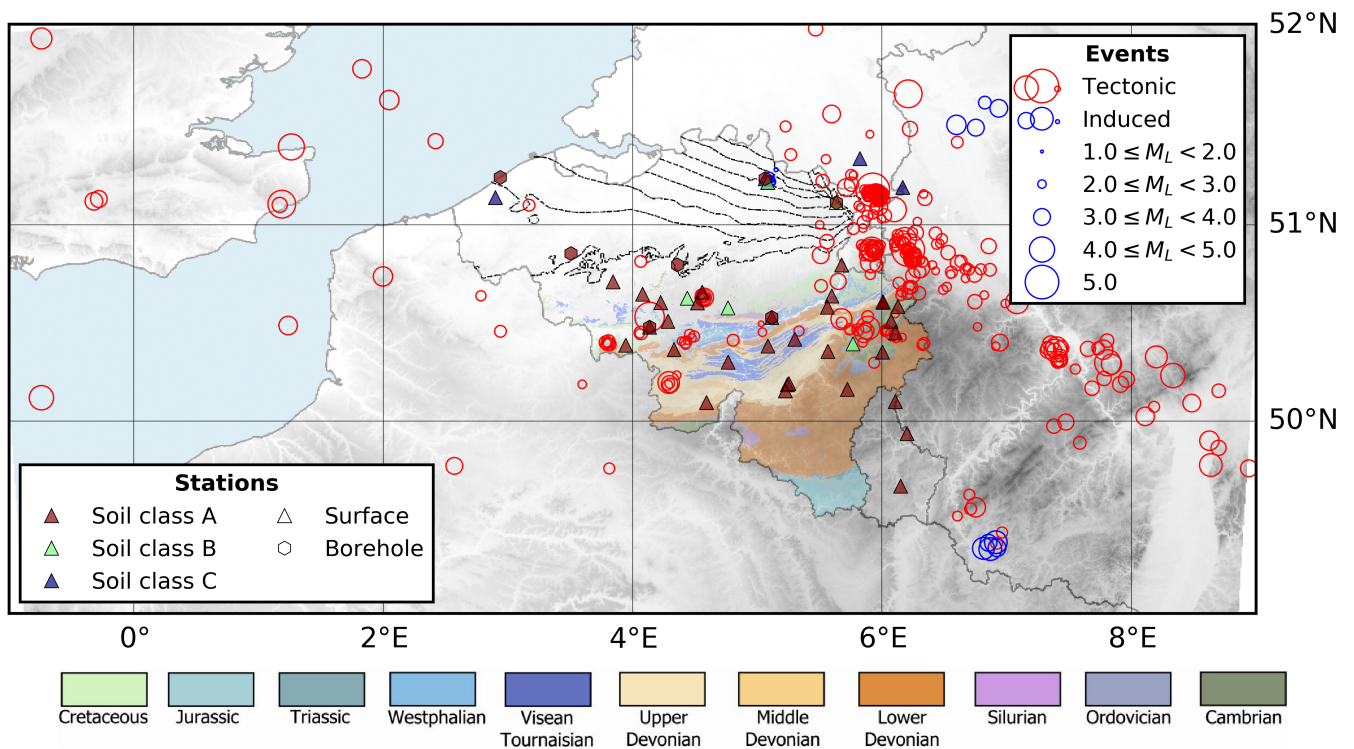


Figure 2 Map showing location of earthquakes included in the BELSHAKE database and seismic stations where at least 10 amplitudes have been measured. Stations are colored by soil class, with different symbols for surface and borehole emplacements. Background image depicts simplified surface geology in Belgium, with bedrock in the south and NNE-ward thickening sediment layers (dashed lines with 100 m contour interval) in the north.

3 Automated amplitude measurement

In the ROB routine practice, half maximum peak-to-peak amplitudes are measured in the S/Lg window on time histories, restituted to displacement according to a custom procedure based on Seidl (1980), and high-pass filtered twice with a corner frequency of 1 Hz and 4th order. This is only possible for velocimetric stations of the Belgian seismic network where the S phase can be picked. Measurements are done manually and are thus time-consuming and operator-dependent. It is not possible to review amplitude measurements because the peak time positions are not available in the ROB catalog.

We first developed code to automate the measurement of displacement amplitudes supporting two different restitution methods and two different peak estimation methods. In this code, waveform data are converted to displacement by removing the instrument response using routines in ObsPy (Megies et al., 2011), followed by single or double integration depending on instrument type (velocimeter or accelerometer, respectively). Subsequently, the signal is either high-pass filtered twice with a corner frequency of 1 Hz and 4th order (restitution method 1), or it is used to simulate a Wood-Anderson instrument with unit sensitivity (restitution method 2). Next, the measurement window is defined (Figure S1 in electronic supplement (ES)), starting at the (manually) picked S-wave arrival time (minus a buffer of 0.1 s) and ending at some maximum duration (30 s) or at the theoretical arrival of the Rayleigh wave, whichever comes first. Finally, the maximum

peak-to-peak amplitude is determined in this window, either between successive peaks (peak method 1, strict peak-to-peak) or taking into account intervening gradient changes (peak method 2, using a Savitzky-Golay filter with a window length of 0.1 s and similar to current manual practice), and divided by two. In contrast to current practice, this procedure can be applied without user intervention to all waveform data for which instrument response is available, including accelerometric data and data from other networks. Additionally, the start and end times of the measured peaks are saved so that they can be reviewed manually if necessary.

Applying the four combinations of restitution and peak measurement methods to all reliable waveform data in the BELSHAKE database, we could calculate vertical amplitudes for ~4960 records from 342 events, roughly three times more than what is available in the ROB catalog for the same time and magnitude range, and additionally horizontal amplitudes for ~3900 records from 330 events. Figure 3 shows histograms of the ratios between the amplitudes obtained with restitution method 1 and peak method 2 and manually determined amplitudes in the ROB catalog for two time intervals. For the most recent period (2015–2022), the agreement is quite good with a mean ratio close to one but a relatively large standard deviation of ~0.7. However, for the period before 2000 we obtain a bimodal distribution with peaks around 0.6 and 1, which suggests that many older values in the ROB catalog are wrong by a factor of ~2 and likely correspond to full peak-to-peak amplitudes. There is no obvious trend with magnitude in either case. The amplitude ratios obtained with the

other methods are shown in Figure S2 in ES; in general, restitution method 2 results in narrower distributions, while peak method 1 results in slightly higher mean values (~ 1.15).

Next, we computed station M_L values corresponding to the automatically determined amplitudes using the original M_L scale (Equations 4 and 5), and event M_L values were computed as the average of the respective station M_L values, excluding outliers (values more than 2 standard deviations away from the initial mean). All results are stored in two tables in the BELSHAKE database, one containing the parameters determined for individual stations and one containing the event solutions. In Figure 4, we compare the recomputed event M_L values with those in the ROB earthquake catalog for restitution method 2 and peak method 2. The results for the other combinations are shown in Figure S3 in ES. These show that all methods slightly underestimate the catalog M_L values. This negative bias is smaller for peak measurement method 2 (-0.05) than for peak method 1 (< -0.1). This suggests that peak method 2 (strictly peak-to-peak) is slightly more robust than the method taking into account inflection points, and hence the former is preferred. The differences between the two restitution methods are not significant. However, there are reasons to prefer method 2 (restitution to Wood-Anderson seismograph): it is limited to ~ 1 Hz, in agreement with the origin of the m_{bLg} scale, and it is also used by neighbouring agencies, e.g. the Bensberg network in Germany (Hinzen et al., 2020). A direct comparison between the catalog M_L values and M_L values obtained with the preferred restitution and peak methods (right panel in Figure 4) shows that most differences appear to be contained within ± 0.3 magnitude units. Outliers with $|\Delta M_L| > 0.35$ are listed in Table 1. One case concerns an event outside of Belgium for which no amplitudes are available in the ROB catalog and the M_L value was likely copied from another agency. For the two events inside Belgium, one has only three amplitudes in the ROB catalog and the other zero. The latter shows the largest absolute difference (~ 1.1), but we consider the newly determined M_L value to be more reliable as 7 out of 9 station M_L values show good mutual agreement (between 1.04 and 1.45).

We thus conclude that the M_L values computed from automatically determined amplitudes are reliable. In addition, not only have we been able to collect more amplitude data using the BELSHAKE database than there are in the ROB catalog, but the better agreement between the M_L values recomputed from these amplitudes and the M_L values in the ROB catalog (Figure 4) compared to the lower agreement between the respective amplitudes (Figure 3) also indicates that recomputed amplitudes and M_L values are more consistent than those in the catalog, particularly for older (pre-2000) events.

4 Recalibration of the Belgian M_L scale

4.1 Recalibration based on the vertical component

Similar to Camelbeeck (1985, 1993), we use magnitude-corrected station amplitudes (in μm), i.e. $\log_{10} A_{\text{station}} + M_{L_Ref} - M_{L_event}$, with M_{L_Ref} equal to $m_{bLg_Ref} = 3$ (Eq. 3) and M_{L_event} calculated with the original scale, to fit a new relation as a function of hypocentral rather than epicentral distance. It is relevant in this context to note that horizontal and vertical location uncertainties in the BELSHAKE database have median values of 0.9 and 1.2 km, respectively, and 95th percentile values of 6.7 and 5.7 km, respectively (Vanneste et al., 2026). Data from some noisy stations (mainly BE.UCCS and BE.BDHN), some accelerometer stations (BE.UCCA, BE.STWA, and BE.MRCA) and from several temporary stations were excluded. The remaining selection includes both bedrock stations (soil class A in Figure 2) and stations situated on soft sediments (soil classes B and C). We only include data from events with at least 5 station amplitudes. To reduce the influence of events determined solely by short-distance stations, we further limit the selection to events with $\geq 66\%$ of contributing stations at distances beyond 30 km. The final selection contains 3985 vertical amplitudes from 264 events, and 2942 horizontal amplitudes from 254 events.

Figure 5a shows the selected magnitude-corrected station amplitudes as a function of epicentral distance, along with the original m_{bLg} relation (Camelbeeck, 1985). The total standard deviation of this relation with respect to the dataset is 0.231. The overall shape of the dataset represents regional attenuation, which we want to capture by fitting, while the vertical variability reflects random variations (including variations due to radiation pattern) and site effects. We tested two relations with respect to hypocentral distance (R_h):

- A relation with linear and logarithmic distance terms, similar to Camelbeeck (1985), but including a reference distance (R_{ref}), as also used in some generic ground-motion prediction equations (e.g., Traversa et al., 2020):

$$\log_{10} A_0 = c_1 + c_2 \log_{10} \left(\frac{R_h}{R_{ref}} \right) + c_3 (R_h - R_{ref}) \quad (6)$$

- A relation similar to the previous one, but with an additional explicit depth (d) term:

$$\log_{10} A_0 = c_1 + c_2 \log_{10} \left(\frac{R_h}{R_{ref}} \right) + c_3 (R_h - R_{ref}) + c_4 \log_{10} d \quad (7)$$

These relations have total standard deviations of 0.230 and 0.229, respectively. Their coefficients are listed in

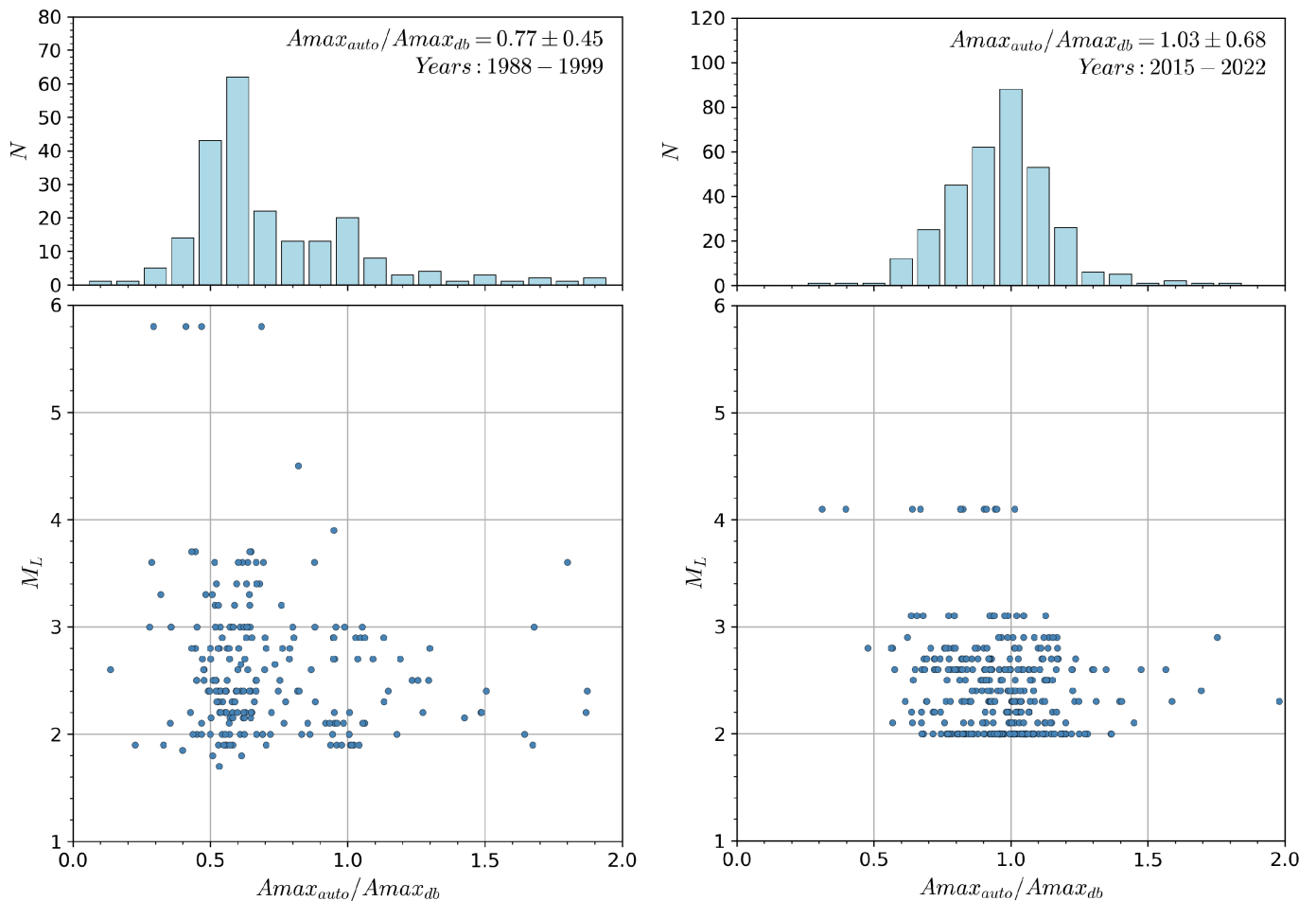


Figure 3 Distribution of ratios between automatically determined amplitudes (restitution method 1, peak method 2, labelled $Amax_{auto}$) and amplitudes in ROB catalog (labelled $Amax_{db}$) for two different periods. Left: 1988 – 1999; Right: 2015 – 2022. Bottom plots show amplitude ratios versus M_L , top plots show histograms.

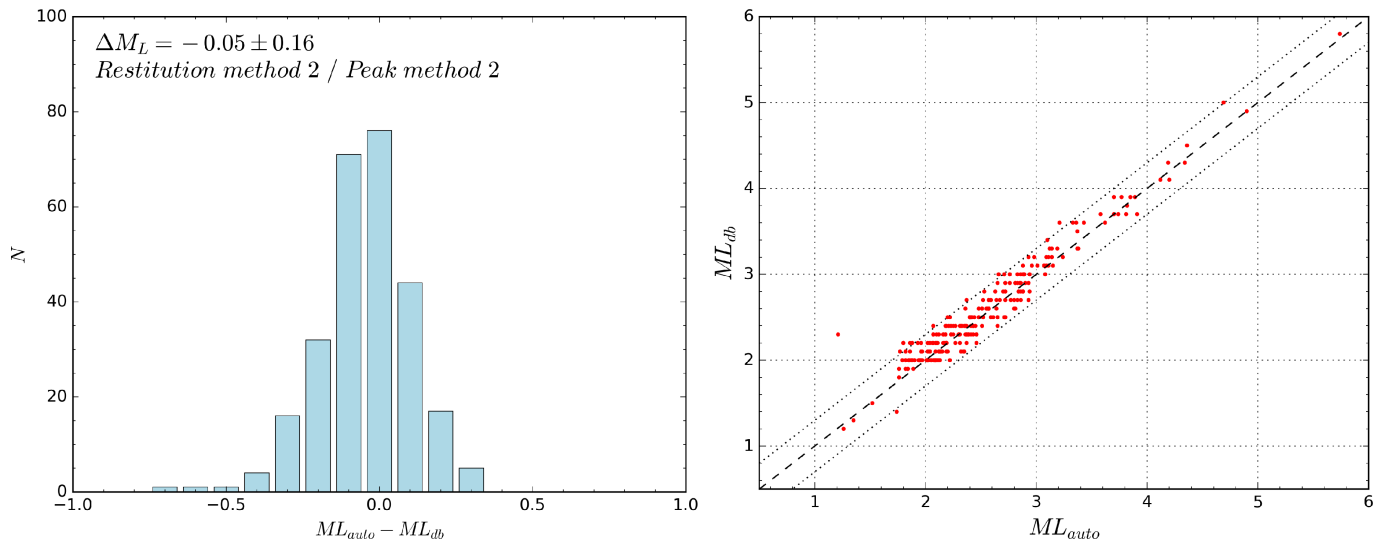


Figure 4 Comparison of M_L values recomputed from automatically determined amplitudes (restitution method 2, peak method 2, labelled $M_{L_{auto}}$) and M_L values in the ROB catalog (labelled $M_{L_{db}}$). Left: Histogram of differences. Right: scatter plot (dashed line: 1:1 relation; dotted line: 1:1 relation ± 0.3 magnitude units). Note that $M_{L_{db}}$ values are available with a precision of only 1 decimal place.

Table 2 and they are shown in Figure 5b and Figure 5c, along with the original m_{bLg} relation adjusted to a focal depth of 10 km. Both relations thus perform similarly to the original relation with respect to epicentral distance.

However, introducing an explicit depth term does not improve the fit, nor is there a visible difference between the fitted curves corresponding to different depths (Figure 5c), in line with the lack of a visible trend with fo-

Date	Name	Lon (°)	Lat(°)	Depth (km)	M_{L_cat}	M_{L_auto}	ΔM_L	n_{cat}	n_{auto}
1997-11-29	VILLMAR (DE)	8.3598	50.3433	4.7	3.6	3.24	-0.36	0	17
2001-07-14	VOERENDAAL (NL)	5.9110	50.8777	1.7	2.2	1.80	-0.40	0	10
2001-09-04	GILLY (BE)	4.4952	50.4337	2.0	2.3	1.21	-1.09	0	9
2008-08-09	OTTIGNIES- LOUVAIN-LA-NEUVE (BE)	4.5733	50.6625	8.2	2.2	1.80	-0.40	3	18

Table 1 List of earthquakes for which absolute difference between catalog (“cat”) M_L and recomputed (“auto”) M_L is > 0.35.

cal depth of the dataset, as shown by the colors in this panel.

If we change to a logarithmic distance scale (Figure 5d), more or less straight sections with different slopes can be discerned. This shape is reminiscent of piecewise continuous geometrical spreading functions with a hinged multilinear shape in log-log space, generalized from Boore (2003) as:

$$F(R) = \begin{cases} \left(\frac{R_0}{R}\right)^{-p_0} & R \leq R_1 \\ F(R_1) \left(\frac{R_1}{R}\right)^{-p_1} & R_1 \leq R \leq R_2 \\ \dots & \dots \\ F(R_n) \left(\frac{R_n}{R}\right)^{-p_n} & R_n \leq R \end{cases} \quad (8)$$

An example is the function used to model Fourier spectra of horizontal S waves in NE America (Atkinson and Boore, 1995; Atkinson and Mereu, 1992; Boore, 2003), which consists of three segments: 1) R^{-1} up to 70 km distance, where the main contribution is from the direct wave; 2) R^0 between 70 and 130 km, where both direct and reflected waves arrive, and 3) $R^{-0.5}$ beyond 130 km (the distance corresponding to the maximum critical angle of all reflected shear waves), corresponding to the Lg wave trapped within the crustal wave guide. The flat middle section implies that a given event produces ground motion at a distance of 130 km with comparable spectral amplitudes as at a distance of 70 km. In the time domain, however, amplitudes could be lower at 130 km due to the effect of increasing duration with distance (Atkinson and Mereu, 1992). In some studies (Altindal et al., 2026), the change in attenuation associated with post-critical reflected phases is modeled using a single hinge distance, but we fitted a piecewise continuous powerlaw model to our dataset by performing a (bounded) minimization to find 3 knickpoint distances and 4 exponents, as well as a vertical offset. The latter is required because unlike a geometrical spreading model, the magnitude-corrected amplitude scale does not start from 1 at the minimum distance. We also allow for an additional knickpoint around ~10 km based on the observed shape of the dataset. The total standard deviation of this powerlaw fit (Figure 5e) is 0.220, which is lower than the previous fits. However, it diverges from the original relation at large distances (> 400 km), mainly because we did not take into account anelastic attenuation. Simple forward modeling indicates that the shape of the attenuation curve is dominated by geometrical spreading, and that the effect of anelastic attenuation should be visible as a slight change in slope of

the near-flat midsection and downward bending of the fourth section. If we include regional anelastic attenuation in the form of a κ gradient (cf. Eq. 2) in the minimization function, as well as some data points matching the original m_{bLg} relation, we obtain the fit in Figure 5f and with parameters listed in Table 3. Its total standard deviation (0.219) is only slightly lower, but it matches the original relation at larger distance. The corresponding κ gradient is 0.34 ms/km (equivalent to $Q_V \cong 840$ for $V_S = 3.5$ km/s), which is less than half of the equivalent value in the original m_{bLg} relation ($\gamma=0.0026$ km $^{-1}$ corresponds to κ gradient of 0.83 ms/km), but still higher than the values obtained in a dedicated study of this parameter for S-waves on the vertical component based on regressions of κ values determined from Fourier spectra (Onvani and Vanneste, 2025). We will further investigate this issue in the calibration based on the horizontal components in the next section.

The calibration formula as a function of hypocentral distance for the zero-magnitude displacement amplitude thus becomes:

$$\log_{10} A_0(R_h) = \log_{10}(A) + y - M_{L_Ref};$$

$$A = G_{R_g;p_g}(R_h) \exp\left(-\frac{\delta\kappa}{\delta R} R_h\right) \quad (9)$$

where G represents the piecewise continuous time-domain geometrical spreading function (Eq. 8) defined by hinge distances R_g and exponents p_g , $\frac{\delta\kappa}{\delta R}$ is the κ gradient, and y the offset, all listed in Table 3. This formula can be used in combination with Eq. 3 to calculate recalibrated M_L values. In the original calibration, Camelbeeck (1985) defined that the reference magnitude of 3.0 corresponds to a maximum displacement amplitude of 2.42 μm at a (epicentral) distance of 10 km. We propose to change the definition of the amplitude for the reference magnitude to a distance that gives similar values in both the old and the new calibrations, using their respective distance metrics. Thus, $M_L = 3.0$ corresponds to a displacement of 0.12 μm at a distance of 200 km. Note that this reference distance has been chosen on purpose beyond the near-flat midsection of the geometrical spreading function.

4.2 Recalibration based on the horizontal components

We determined amplitudes on the horizontal components in a similar way as for the vertical component. For

Fit	Rref (km)	C ₁	C ₂	C ₃	C ₄	σ (s)
Eq. 6	0.12876	0.00012	-1.22119	-0.00030	-	0.230
Eq. 7	0.12207	0.00246	-0.00025	-1.23435	0.05629	0.230

Table 2 Fitted coefficients for two tested relations against magnitude-corrected time-domain amplitudes of S/Lg waves around Belgium measured on the vertical component.

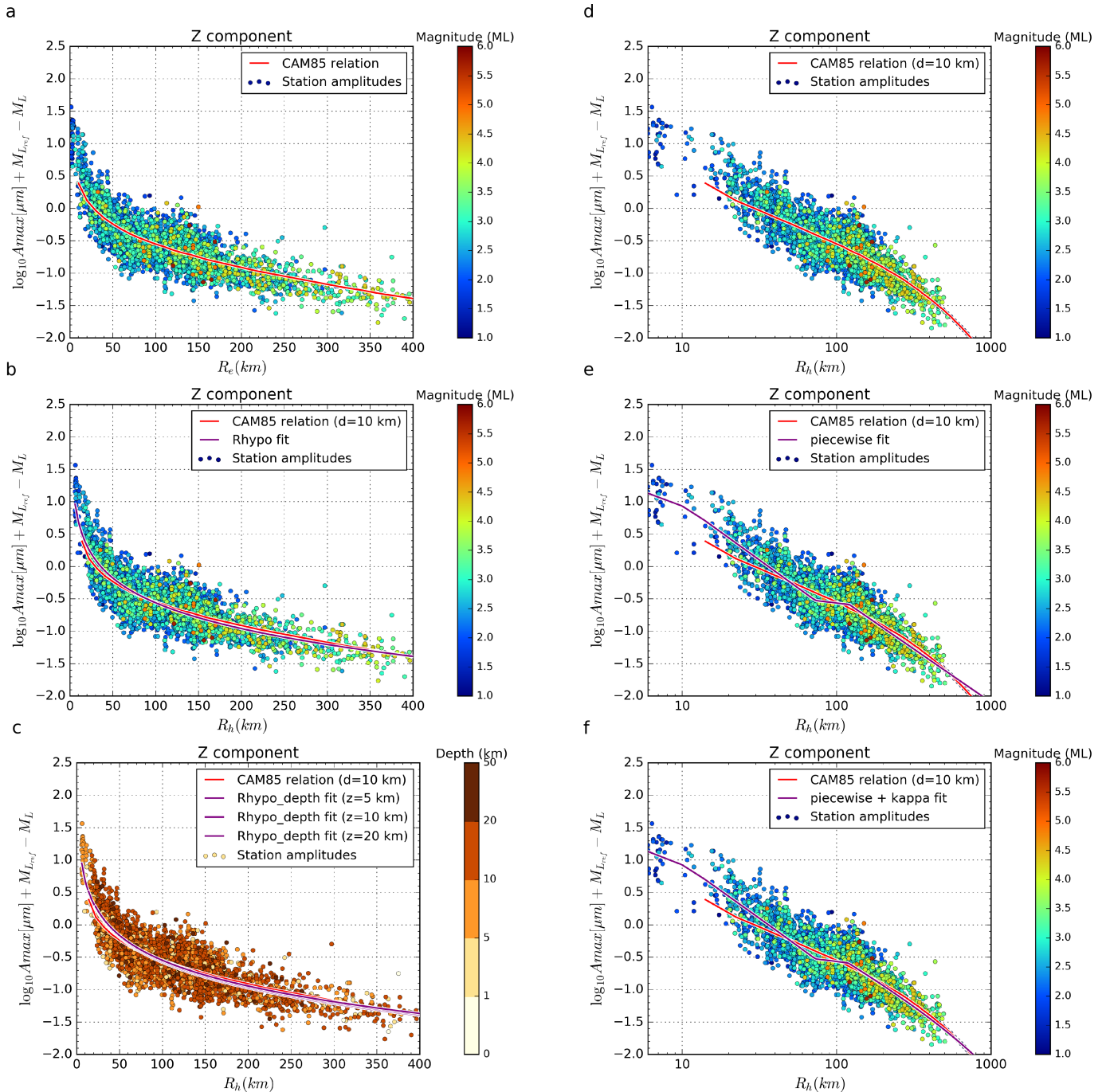


Figure 5 Selected magnitude-corrected station amplitudes (measured on the Z component) versus distance, along with different fits. a) Original m_{bLg} relation versus linear epicentral distance (R_e); b) “Rhyppo” fit (Eq. 6) versus linear hypocentral distance (R_h); c) “Rhyppo_depth” fit (Eq. 7) for depths of 5, 10 and 20 km versus linear R_h ; d) Original m_{bLg} relation (adjusted to a focal depth of 10 km) versus logarithmic R_h ; e) Piecewise continuous powerlaw fit versus logarithmic R_h ; f) Piecewise continuous powerlaw + κ gradient fit versus logarithmic R_h . Amplitudes are color-coded according to event M_L values, except in c), where colors correspond to event depths.

each station, the two horizontal components were combined in a single value by taking the geometric mean

(GM), ignoring stations having only 1 usable horizontal component. A similar fit (piecewise continuous power-

Comp.	offset	R_0	p_0	R_1	p_1	R_2	p_2	R_3	p_3	$\frac{\delta\kappa}{\delta R}$	σ
Z	1.851	1.0	-0.918	10.8	-1.678	72.8	-0.202	119.0	-1.367	3.444E-4	0.219
GM	2.156	1.0	-0.746	9.5	-2.10	62.0	-0.0107	110.0	-1.405	4.814E-4	0.259

Table 3 Parameters (hinge distances and exponents of different segments, overall offset and κ gradient) of piecewise continuous powerlaw + κ gradient fit to magnitude-corrected time-domain amplitudes of S/Lg waves around Belgium on the vertical (Z) and horizontal (GM) components. The corresponding covariance matrices can be found in Table S1 (Z) and Table S2 (GM) in ES.

law + κ gradient) was then performed (Table 3), which is shown in Figure 6a along with the M_L scale of the Bensberg network in Germany (Hinzen et al., 2020), which is also based on horizontal components. However, the total standard deviation (0.259) is significantly higher (> 15%) than that obtained for the Z component. Although the use of horizontal components thus does not provide any advantage over the Z component, this result is important because it allows comparing with existing S-wave attenuation models, which are generally available for the horizontal component only. Compared to the earlier mentioned geometrical spreading model for NE America (Atkinson and Boore, 1995; Atkinson and Mereu, 1992; Boore, 2003), our solution has an additional segment at short distance (< ~10 km), the flat midsection is situated at shorter distance, and the absolute powerlaw exponents are higher. The latter is partly explained by the difference between time-domain amplitudes used to determine M_L and amplitudes used to model Fourier spectra. According to Parseval's theorem, root-mean-square amplitudes in the time domain (A_{rms}) should be related to Fourier amplitudes through the window length (i.e. duration) T (Taylor et al., 2002):

$$A_{rms} \propto T^{-0.5} A_{Fourier} \quad (10)$$

Numerical modelling of 1-Hz Lg geometrical spreading (Yang, 2002) indicates that peak-to-peak amplitudes (A_{PP}), on which the Belgian M_L scale is based, decay with a similar exponent as rms amplitudes and confirms that this exponent is about twice as high as the Fourier-domain exponent. Thus, if duration is proportional to distance R^t , it follows that:

$$A_{PP} \propto R^{-0.5t} A_{Fourier} \quad (11)$$

and the geometrical spreading exponents in the time (n_{PP}) and Fourier ($n_{Fourier}$) domain (taking into account their sign) should be related as:

$$n_{PP} = n_{Fourier} - 0.5t \quad (12)$$

In the BELSHAKE database, durations have been computed following different metrics, including D5-95 (Trifunac and Brady, 1975), which is one of the most frequently used parameters of seismic duration and corresponds to the buildup from 5% to 95% of the cumulative normalized Arias Intensity (Arias, 1970). These have been converted to path durations by subtracting the corresponding source durations, which are approximated as the inverse of the corner frequencies of the respective events. The latter were estimated simultaneously with the seismic moments (see §6). Figure 6b

shows the results for the horizontal (GM component) S wave recorded by stations situated on rock. Similar to the magnitude-corrected amplitudes, a log-log plot of these durations suggests a piecewise continuous powerlaw with a flat midsection bounded by knickpoints at more or less the same distances. We therefore performed a powerlaw fit to the data, similar to that for the amplitude data, but with an additional knickpoint at larger distance (~300 km) instead of around 10 km. This fit, shown by the red line in Figure 6b and with coefficients listed in Table 4, can be considered as a path duration model for horizontal S/Lg waves around Belgium. It can be seen that durations are in between those given by the models for western North America (WNA, Boore and Thompson, 2014) and eastern North America (ENA, Boore and Thompson, 2015). A similar powerlaw model could also be fitted for the Z component (Table 4).

With this duration model, we could compute geometrical spreading exponents in the Fourier domain by correcting (Eq. 12) the time-domain spreading exponents with the duration exponents for the same segments. Compared to the ENA spreading model (Atkinson and Boore, 1995), the slopes of the sections flanking the flat midsection are steeper (Figure 7a). We also find that the value of the κ gradient (0.48 ms/km, equivalent to $Q_H \cong 600$ for $V_S = 3.5$ km/s) given by the fit is higher than the values obtained in a dedicated study of this parameter for horizontal S-waves (Onvani and Vanneste, 2025), similar to what was observed for the Z component (§4.1). To try and improve these discrepancies, we tested another way to obtain Fourier-domain spreading exponents by directly integrating duration (path duration from the model + source duration from corner frequency) into the amplitude fitting procedure (through Eq. 10). Doing so, the fits (not shown here) are visually indistinguishable, but the κ gradients (Table 5) are reduced to 0.18 ms/km (GM, $Q_H \cong 1590$) and to 0.10 ms/km (Z component, $Q_V \cong 2860$), which are in better agreement with the results of the dedicated study. The powerlaw exponents describing the geometrical spreading are also slightly lower (Figure 7a, Table 5), but still higher (in absolute terms) than those in the ENA model, indicating stronger attenuation through geometrical spreading in our region. On the other hand, a similar segmented spreading model developed for the United Kingdom (Rietbrock and Edwards, 2019) shows an even higher exponent for the segment beyond the flat midsection (Figure 7), although the authors acknowledge that this may capture frequency-dependent Q , which they did not model.

To test this, we try reproducing the time-domain at-

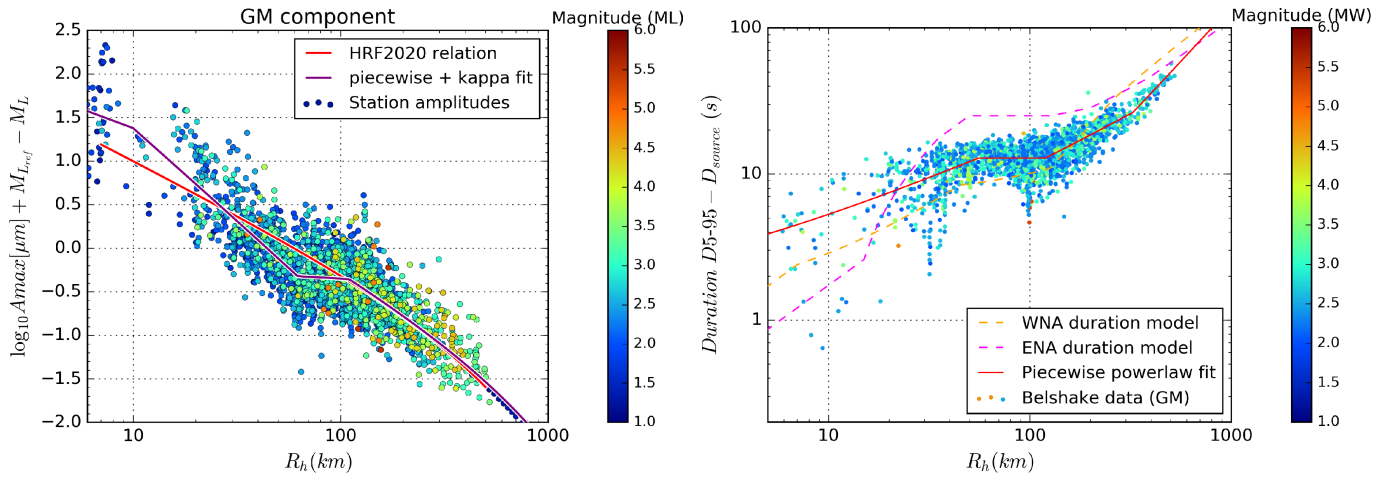


Figure 6 Left: Selected magnitude-corrected station amplitudes (GM component) versus R_h , along with piecewise continuous powerlaw + κ gradient fit. Right: path durations (D5-95 corrected for source duration) for horizontal S waves recorded at rock stations in the BELSHAKE database, along with piecewise continuous powerlaw fit and two models used in the USA (WNA: Boore and Thompson, 2014; ENA: Boore and Thompson, 2015).

Comp.	offset	R_0	p_0	R_1	p_1	R_2	p_2	R_3	p_3	α
Z	1.6987	1.0	0.615	59.7	0.	119.4	0.692	315.3	1.546	2.649
GM	1.2802	1.0	0.610	56.1	0.	119.2	0.772	323.9	1.50	2.649

Table 4 Parameters (hinge distances and exponents of different segments and overall offset) of fitted piecewise continuous powerlaw models for path durations of S/Lg waves around Belgium on the vertical (Z) and horizontal (GM) components.

tenuation model that we derived from our M_L calibration, by stochastic simulation (Boore, 2003) based on the inferred Fourier spreading models and corresponding κ gradients, in combination with the path duration model. We simulate random acceleration time series for a range of distances (50 samples at each distance) with fixed source and site parameters, convert them to displacement on a Wood-Anderson seismograph, determine (half) the maximum peak-to-peak amplitudes, and take the average at each distance. The results (Figure 7b) validate the inferred relation between time-domain and Fourier amplitudes and illustrate that integrating the duration model in the fitting procedure yields slightly better results than correcting time-domain spreading exponents afterwards. For comparison, we also show a simulation based on the ENA spreading and duration models (Atkinson and Boore, 1995). Even though this result does not take into account anelastic attenuation, it is clear that it strongly diverges from our observations.

Finally, our spreading model includes an additional knickpoint at ~ 10 km compared to the ENA and UK models. It is not so well constrained by the data, however: the standard deviation is large (see Table S2 in ES), because there are few data points between 8 and ~ 15 km and the variability at the shortest distances is relatively high (Figure 6a). The latter is probably due to the high corner frequencies of the small events in this distance range. It is not clear if the change in slope is related to an actual change in geometrical spreading or to stronger anelastic attenuation at shorter distances, which is expected because focal depths become necessarily shallower. However, a similar knickpoint was found for the

vertical component, so it is likely not an artefact in the data. We hope to gain more insight in this characteristic when more data become available.

Based on the fit with incorporated duration, the calibration formula for the zero-magnitude displacement can be written equivalently to Eq. 9 as:

$$A = G_{R_g; p_g}(R_h) \exp\left(-\frac{\delta\kappa}{\delta R} R_h\right) [D_{R_d; p_d; y_d}(R_h)]^{-0.5}$$

$$\log_{10} A_0(R_h) = \log_{10}(A) + y - m_{L_{Ref}} \quad (13)$$

where G represents the piecewise continuous geometrical spreading function defined by hinge distances R_g and exponents p_g in Table 5, D the duration model defined by hinge distances R_d , exponents p_d and offset y_d in Table 4, and $\frac{\delta\kappa}{\delta R}$ the κ gradient and y the overall offset in Table 5.

While Equations 9 and 13 yield results that are visually indistinguishable, the differences between them reflect the relation between time-domain and Fourier amplitude decay with distance. The main importance of Eq. 13 is that the geometrical spreading function and κ gradient in it can be used to model elastic and (regional) anelastic attenuation of horizontal S-wave Fourier spectra. This is useful for determining moment magnitudes from displacement spectra, and in combination with the path duration model for S waves also for stochastic simulation of acceleration time histories (Boore, 2003).

4.3 Residual analysis

We calculated station M_L residuals by subtracting the respective event M_L values (if constrained by three or

Comp.	offset	R_0	p_0	R_1	p_1	R_2	p_2	R_3	p_3	$\frac{\delta\kappa}{\delta R}$	σ
Z	2.00025	1.0	-0.683	10.9	-1.478	74.0	-0.306	119.0	-1.163	1.00E-4	0.220
GM	2.27455	1.0	-0.490	9.5	-1.885	62.2	-0.135	110.0	-1.229	1.78E-4	0.259

Table 5 Parameters (hinge distances and exponents of different segments, overall offset and κ gradient) of piecewise continuous powerlaw + κ gradient fit (directly incorporating duration model from Table 4) to magnitude-corrected time-domain amplitudes of S/Lg waves on the vertical (Z) and horizontal (GM) components. Note that distances and exponents represent Fourier-domain geometrical spreading model for S/Lg waves around Belgium.

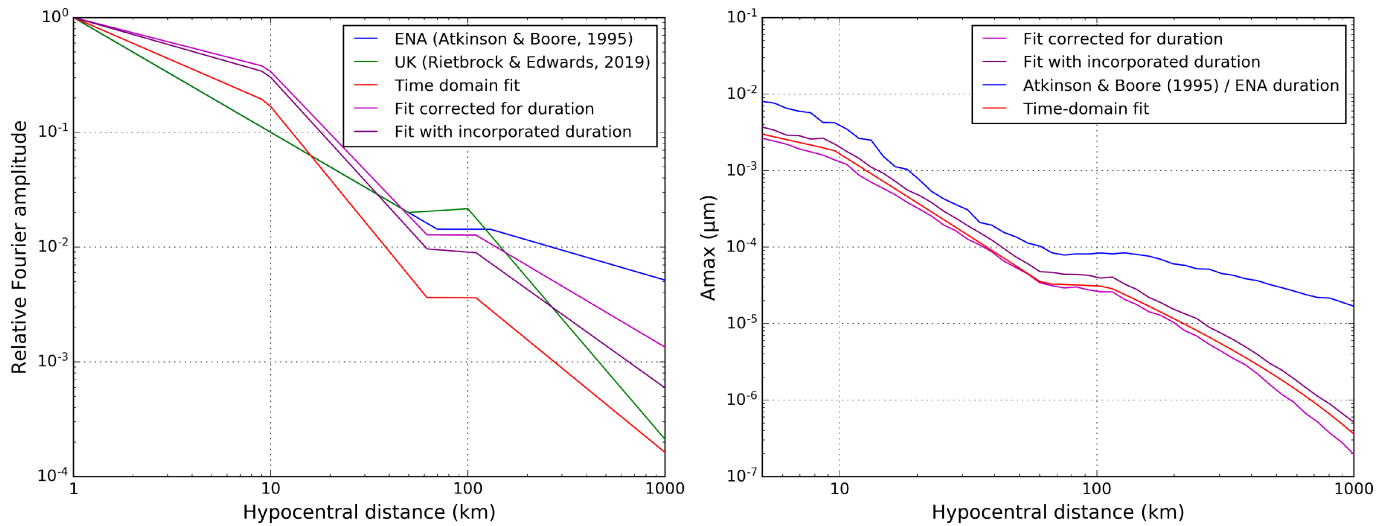


Figure 7 Left: Comparison of geometrical spreading model for eastern North America (Atkinson and Boore, 1995) and UK (Rietbrock and Edwards, 2019) with fitted spreading models for horizontal S-waves obtained in this study; note that all models are Fourier models, except for time-domain fit. Right: Comparison of M_L -derived time-domain attenuation model (geometrical spreading + anelastic attenuation) for horizontal S waves with simulated time-domain amplitudes based on different fits and the BELSHAKE path duration model. Note that M_L -derived time-domain model concerns relative amplitudes and has been shifted vertically to lie closer to the simulated curves.

more stations) from the station M_L values. The mean and standard deviation for each station are listed in Table S3 in ES. Residuals range between -0.3 and +0.6 for the Z component, and between -0.3 and +0.7 for the GM component. To determine whether there is any relation between these station M_L residuals and site conditions, we plot them as a function of the average shear-wave velocity in the top 30 m (V_{S30}) in Figure 8. The correlation appears to be stronger for the GM component than for the Z component: GM residuals are generally negative for stations with $V_{S30} > 1250$ m/s, and increase more or less linearly from this V_{S30} value downward. This indicates a higher sensitivity of horizontal amplitudes to site effects, with negative residuals for (hard) rock sites and positive residuals for standard rock and loose soil sites. Next, the mean station M_L residual was subtracted from the magnitude-corrected amplitudes for stations where at least 10 events were recorded. The results for both Z and GM components are shown in Figure S4 in ES. The total standard deviations were thus further reduced to 0.183 and 0.202, respectively. Although the relative improvement is stronger for the GM component, both the total standard deviation and the variability at short distances are still significantly smaller for the Z component.

We also evaluated whether the individual M_L residuals show any trend with respect to different parameters

(hypocentral distance, earthquake depth, event magnitude and V_{S30}), with and without correcting for the station residuals derived above. The results are shown in ES in Figure S5 for the Z component, and in Figure S6 for the GM component. Only for V_{S30} , uncorrected residuals show a trend, but this disappears after station correction. All other parameters do not show any discernible trend, indicating that the new calibration is not biased with respect to the tested parameters. Notably, compared to the original scale (Figure S7 in ES), residuals have clearly improved with respect to distance, particularly at short distance. The overall average residual is close to zero, and the overall standard deviation is reduced from 0.22 to 0.18 for the Z component, and from 0.25 to 0.19 for the GM component. These values are similar to the standard deviation of 0.2 obtained for the new local magnitude model of Switzerland (Altin et al., 2026), illustrating the good performance of our new calibrations.

5 Application of the recalibrated M_L scale

We implemented the recalibration parameters in our M_L calculation code, adding an option to choose between different calibrations/models and components. To adopt the new recalibrated M_L scale in practice, we

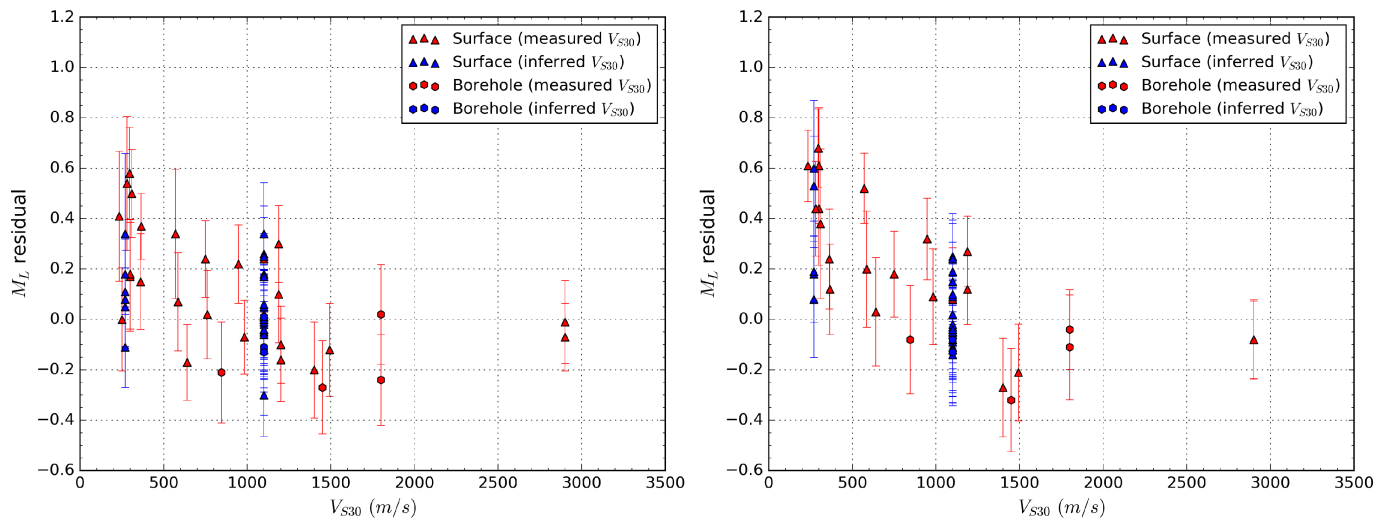


Figure 8 Average M_L residuals for stations with at least 10 amplitude measurements versus V_{s30} (average shear-wave velocity in the top 30 m), either measured or inferred from the soil class. Left: Z component. Right: GM component.

recommend applying Eq. 9 with the coefficients in Table 4 to half maximum peak-to-peak amplitudes on the Z component simulated for a Wood-Anderson seismograph with unit sensitivity. We applied this procedure to two sets of earthquakes: the BELSHAKE database (which contains only few earthquakes with $M_L < 2.0$) and earthquakes with $M_L \leq 2.0$ from the ROB catalog. While the former were used in the recalibration, the latter were not. The results (limited to those based on at least 3 stations) are compared to the M_L values calculated with the original scale (Eq. 4 & 5). For the first set (Figure 9, top panel), the agreement between both M_L estimates is very good, with zero bias and very low variability (standard deviation ~ 0.04). This is both expected and intended, as most earthquakes in BELSHAKE have been recorded by stations over a wide range of distances, and the recalibrated M_L scale should give similar results to the original scale in this case. For the second set (Figure 9, middle panel), the variability is clearly higher and there is a tendency for increasing negative bias (recalibrated $M_L < \text{original } M_L$) with lower M_L , which is expected as the contribution of stations at longer distances gradually drops. The largest outlier in this plot corresponds to a shallow induced earthquake which was recorded only at epicentral distances ≤ 6 km. In principle its M_L is undefined on the original scale, but would be 1.5 if extrapolation is allowed. With the recalibrated scale, which is defined for this distance range, this value is reduced to 0.8. This compares well to the catalog value (0.9), which has been determined relatively to a larger earthquake in the sequence.

Finally, we also compare the recalibrated M_L values based on the Z component and the geometric mean. Figure 9 (bottom panel) shows that the bias is negligible, but slightly increases with decreasing magnitude. All differences are constrained within ± 0.3 magnitude units, except for one earthquake. Inspection shows that in this case, there is a relatively higher contribution of short-distance stations for the GM component, and that the GM amplitudes in these stations are all higher than the mean plus one standard deviation, resulting in

$$M_L(GM) > M_L(Z).$$

To further evaluate the performance of the recalibrated M_L scale, we compare observed and predicted amplitudes versus distance for individual earthquakes. We demonstrate this using two examples: a $M_L = 3.1$ earthquake near Kinrooi (Belgium) in 2018 and a $M_L = 1.5$ earthquake near Liège (Belgium) in 2025. The latter event was not used in the recalibration and was recorded by several stations at short distance. Figure 10 shows the plots for the original scale and the recalibrated scales based on both Z and GM components. Note that all models yield the same M_L to 1 decimal precision for both events. For the first event, both scales seem to perform equally well, even for amplitudes recorded at epicentral distances around 10 km. The second event shows much larger discrepancies between amplitudes recorded at epicentral distances < 10 km and the original scale, whereas the recalibrated scale shows a much better fit. The main difference between both cases is the focal depth, ~ 19 km for the first event versus < 4 km for the second event. As a result, hypocentral distances are significantly smaller for the second event, and these are better taken into account in the recalibrated scale.

These results demonstrate that we have successfully recalibrated the Belgian M_L scale based on hypocentral rather than epicentral distance, extending its range of validity to shorter distances ($> \sim 6$ km) while remaining consistent with the original scale, and with improved residuals.

6 Relation with moment magnitude

In a parallel study (Vanneste et al., 2026), we also calculated moment magnitudes (M_W) for earthquakes in the BELSHAKE database using the python package SourceSpec (<https://github.com/SeismicSource/sourcespec>) (Satriano et al., 2016; Satriano, 2022). For each earthquake, two different calculations were performed: one using the SH-wave on the transverse component (if available), in combination with the new segmented spread-

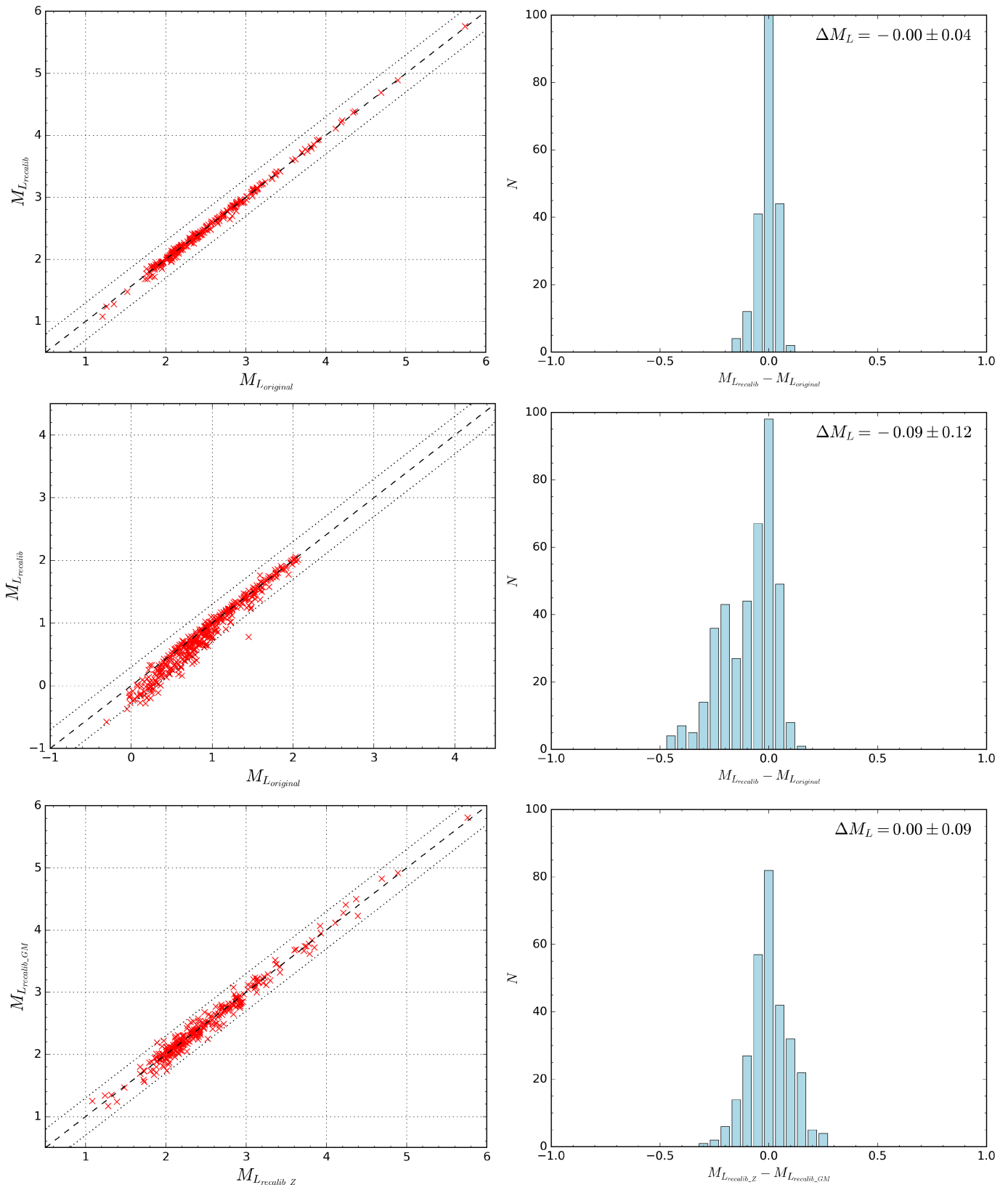


Figure 9 Comparison of M_L values calculated according to different scales with at least 3 contributing stations. Top: Recalibrated M_L values vs M_L values calculated with the original scale based on the Z component for earthquakes in the BELSHAKE database (M_L mostly ≥ 2.0). Middle: Same as above for earthquakes with $M_L \leq 2.0$ in the ROB catalog. Bottom: recalibrated M_L values based on the Z component versus based on the geometric mean for earthquakes in the BELSHAKE database. Left: scatter plot (dashed line: 1:1 relation; dotted line: 1:1 relation ± 0.3 magnitude units). Right: histogram of residuals.

ing model derived in this study, and a second one using the P-wave on the vertical component. As we don't have a specific geometrical spreading model for P-waves, we

had to resort to a simple R^{-1} geometrical spreading model. The final M_W value was calculated as the average (weighted by the number of contributing stations)

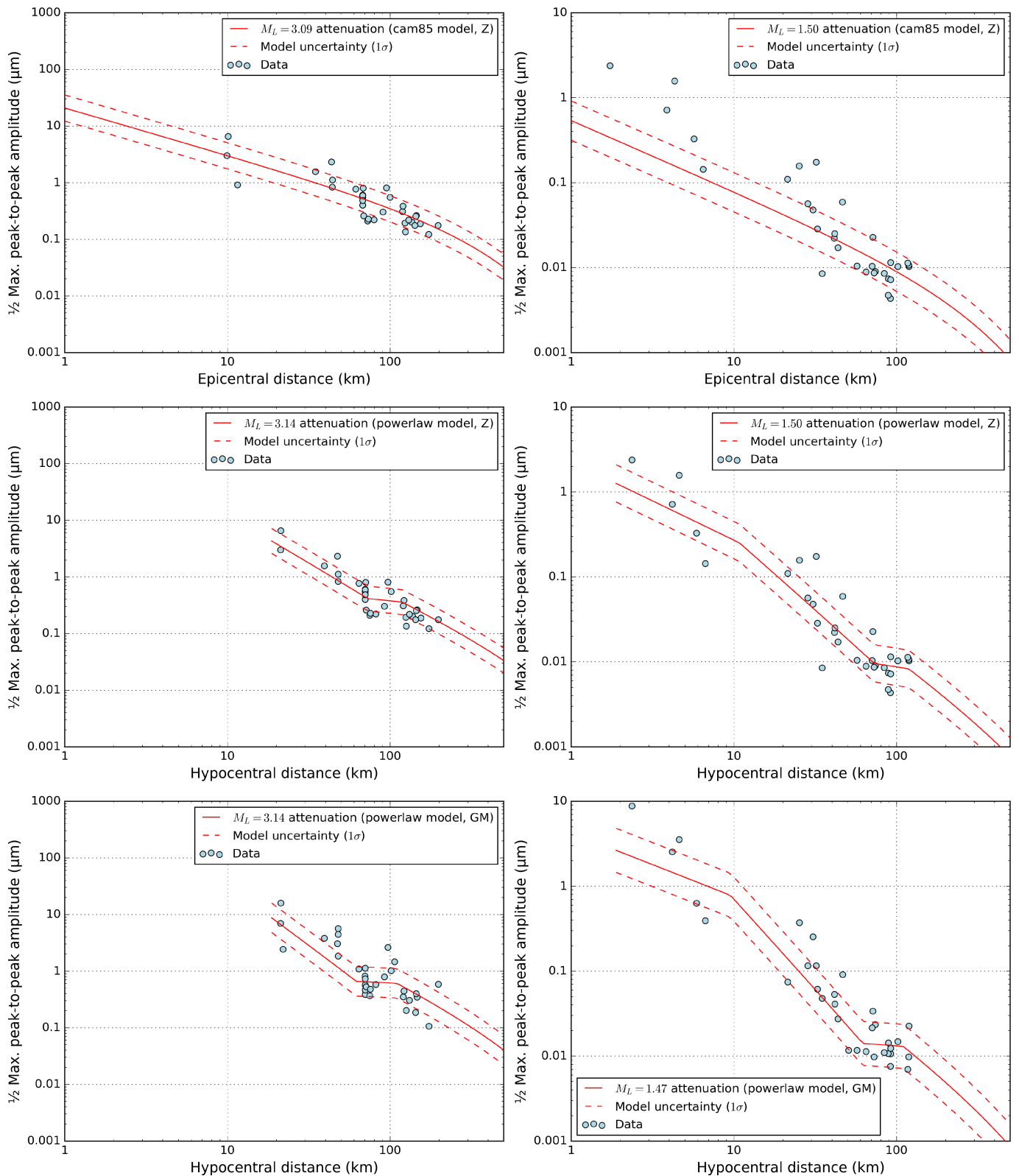


Figure 10 Comparison of observed and predicted amplitudes versus distance for 2 events and 3 different M_L calibration models. Left column: $M_L = 3.1$ Kinrooi earthquake in 2018 (~ 19 km focal depth). Right column: $M_L = 1.5$ Liège earthquake in 2025 (< 4 km focal depth). Top row: original (“cam85”) M_L scale. Middle row: recalibrated (“powerlaw”) M_L scale based on the Z component. Bottom row: recalibrated M_L scale based on the geometric mean. Note different distance metric for original and recalibrated scales.

of the two solutions. In both cases, we used average radiation coefficients, but the use of both P- and S-waves should minimize any biases related to the radiation pattern.

With the improved M_L and M_W estimates in the

BELSHAKE database, it is possible to analyze the relation between both scales (Figure 11). We performed two fits (see Table 6 for coefficients and uncertainties) using orthogonal distance regression (as implemented in the `scipy.odr` Python module, Virtanen et al., 2020) to

account for the uncertainty in both M_L and M_W : 1) a second-order polynomial relation with the same form as the empirical relation of [Hinzen et al. \(2020\)](#) based on the HRF2020 catalog for the northern Rhine area, and 2) a linear fit of the form $M_W = \frac{2}{3}M_L + C$, which is a theoretical relation based on random vibration theory that should be valid for small earthquakes with $M_W < \sim 4$ ([Munafò et al., 2016](#)), although [Deichmann \(2017\)](#) notes that the change in scaling may occur in the $2 < M_W < 4$ range, depending on the data set and geological setting. Based on the shape of the dataset, we imposed an upper limit of $M_L = 4.5$ for the linear fit, while the lower limit was $M_L = 1.6$ in both cases. The quadratic term in the first fit is close to zero, and the difference between both fits is small, both implying a crossover point (where $M_L = M_W$) near $M_L = 2.7$. This indicates that our results are in good agreement with theoretical expectations, and also demonstrates that these empirical relations are robust and can be applied to earthquakes for which we have determined M_L (using the same method and scale!) but are not able to determine M_W directly. However, neither of these relations is valid for $M_L > \sim 4.5$.

In the right panel of Figure 11, the quadratic fit is compared to two existing empirical relations that are relevant for Belgium: a relation by [Grünthal et al. \(2009\)](#) based on the European CENEC catalog, and the above-mentioned HRF2020 relation ([Hinzen et al., 2020](#)). The former is valid for $M_L \geq 3.5$, the latter for the range $-0.7 \leq M_L \leq 4.6$. Compared to the HRF2020 relation, the new fit predicts slightly higher M_W for $M_L > \sim 2.25$. The difference with the [Grünthal et al. \(2009\)](#) relation is larger, the latter predicting significantly higher M_W for $M_L > \sim 3.8$, and lower M_W below this threshold. For higher M_L values, the [Grünthal et al. \(2009\)](#) relation is likely more reliable, but it is clear that it should not be applied to convert M_L values below ~ 3.0 .

Quadratic fit				Linear fit	
a_0	a_1	a_2	σ	C	σ
0.620	0.795	-0.00692	0.106	0.893	0.102

Table 6 Coefficients and total standard deviations of fitted M_L -to- M_W relations derived from the BELSHAKE database, based on orthogonal distance regression. The covariance matrix of the quadratic fit can be found in Table S4 in ES. The uncertainty on coefficient C of the linear fit is 0.006135.

7 Conclusions

Based on automated S-wave amplitude picking on reliable displacement waveforms in the BELSHAKE database, we have recalibrated the Belgian M_L scale by fitting magnitude-corrected station amplitudes as a function of hypocentral distance. To reduce the influence of magnitudes constrained solely by nearby stations, we only selected earthquakes with a majority of contributing stations farther than 30 km. Several relations were tested, but the best fit was obtained with

a 4-segment piecewise continuous powerlaw function (hinged quadri-linear shape in log-log space) representing time-domain geometrical spreading, in combination with an exponential function representing regional anelastic attenuation (κ gradient). This spreading function features an almost flat section between ~ 60 -70 km and ~ 110 -120 km, which is similar to published Fourier spreading models for eastern North America ([Atkinson and Boore, 1995](#)) and the UK ([Rietbrock and Edwards, 2019](#)).

In addition to the Z component used in the original scale, we also performed a similar calibration for the horizontal components (combined into the geometric mean). Although the M_L scales of some neighbouring networks are also based on horizontal displacements, we find that this calibration carries a significantly larger uncertainty. We therefore recommend to continue using the Z component for defining the Belgian M_L scale. On the other hand, the results obtained for the horizontal components are interesting because they can be compared to published geometrical spreading and anelastic attenuation models, which predominantly concern horizontal S waves. This requires a better insight in the relation between Fourier and time-domain attenuation, which we obtained by evaluating S-wave path durations based on D5-95 data in the BELSHAKE database, corrected for source durations. We were also able to fit a piecewise continuous powerlaw function to these data. By directly incorporating this duration model in the fitting procedure, we obtain a Fourier geometrical spreading model with smaller absolute exponents than the time-domain model, which are closer to (but still mostly higher than) the exponents of the aforementioned published models. It is worth noting that our duration model displays a flat section in a similar distance range (~ 55 to ~ 115 km) as the geometrical spreading model, implying that there is almost no attenuation in both the time and Fourier domains over this distance interval. One feature of the spreading model that is not well understood yet is the additional knickpoint at ~ 10 km compared to models published for other regions. We hope that continued data collection will allow determining if this is related to an actual change in geometrical spreading or if it could be an effect of stronger anelastic attenuation at shorter distances (and hence shallower crustal layers), which cannot be modelled as a simple function of distance.

We applied the recalibrated M_L scale to two sets of earthquakes, with (original) M_L ranging between 0.0 and 5.8, and compared these to the values given by the original scale. We observe little difference (zero bias and low variability) for earthquakes with $M_L \geq \sim 1.5$, which are generally recorded over a broad range of distances. In this case, the recalibrated scale is thus consistent with the original scale, as intended. For smaller earthquakes ($M_L < \sim 1.0$), the discrepancies are larger and tend to increase with lower M_L , which is expected as the contribution of stations at longer distances gradually drops. In this case, the original scale overestimates the real M_L value, while the recalibrated scale specifically addresses this by considering hypocentral rather than epicentral distance, thus indirectly taking

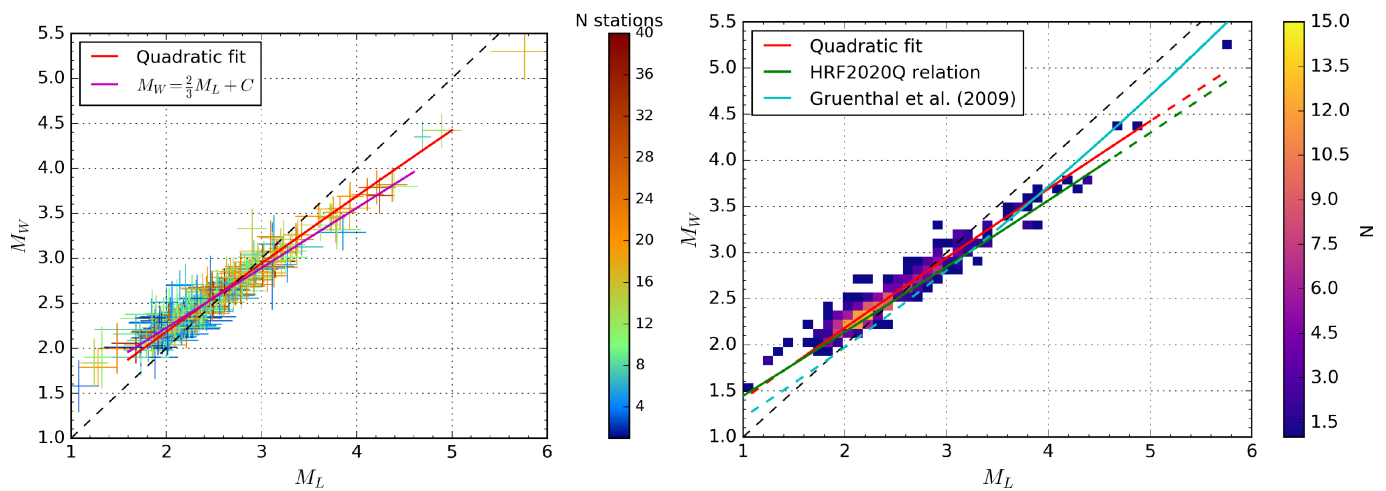


Figure 11 Plot of local magnitudes versus moment magnitudes in the BELSHAKE database. Left: individual data points with error bars and colored by the number of contributing stations, compared to two fitted M_L - M_W relations. Right: density plot, comparing quadratic fit with two existing empirical relations (Grünthal et al., 2009; Hinzen et al., 2020). Dashed colored lines indicate range where corresponding empirical relations are extrapolated; black dashed line represents 1:1 relation.

into account focal depth. This is also demonstrated by comparing observed amplitudes and predicted attenuation curves for individual earthquakes. In addition, the range of validity of the new scale extends to shorter distances ($> \sim 6$ km) and has improved residuals compared to the original scale. We also determined station residuals, which allow some further reduction of the uncertainty by ~ 0.036 magnitude units.

The recalibrated M_L scale will mainly improve magnitude determination for small earthquakes recorded by the Belgian seismic network, and should be applied retroactively to digitally recorded earthquakes in the ROB catalog since ~ 1985 . More accurate M_L determination is also of great value for recently installed and planned local networks monitoring geothermally induced seismicity in Belgium, which are operating traffic-light systems based on small earthquakes. The automated amplitude picking developed in this study also allows interactive review and correction if necessary, which will facilitate and speed up routine M_L estimation of future earthquakes recorded by the Belgian seismic network. Furthermore, the geometrical spreading and duration models for horizontal S waves that we derived in this study are important for many other seismological applications, including determining moment magnitudes (M_W) from displacement spectra, modeling of Fourier spectra, and stochastic simulation of earthquake ground motion.

Finally, we also fit an empirical relation between M_L and M_W , the calculation of which was partly based on the new geometrical spreading model for horizontal S-waves derived in this study. This relation is in good agreement with the one established by the neighbouring Bensberg network in Germany, as well as with theoretical considerations, and allows reliable estimation of M_W for earthquakes for which we have determined M_L , but are not able to determine M_W directly.

The recalibrated Belgian M_L scale should not be regarded as an entirely new scale, as it is still based on the original scale of Camelbeek (1985). Compared to the

original scale, it has been adjusted for short distances and expressed as a function of hypocentral rather than epicentral distance, and the relative contribution of geometrical spreading and anelastic attenuation has been refined. A further improvement that may be envisaged in the future would be to establish a new M_L scale by performing a joint inversion of all event and station data at once, e.g. using mixed-effects regression similar to Altindal et al. (2026) in Switzerland. The anelastic attenuation parameters determined in this study (κ gradient and equivalent Q value) are less well constrained and it is not clear if they are also influenced by higher frequencies than 1 Hz; this should be corroborated by analyzing full spectra, e.g. using the generalized inversion technique (e.g., Grendas et al., 2021).

Acknowledgements

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Data and code availability

All station amplitudes and M_L values calculated in this study are available on Zenodo (Vanneste et al., 2025). An extract of the BELSHAKE database has been released as a flatfile (plain text table containing ground-motion

data and associated event and station metadata for each record) on Zenodo (Vanneste and Onvani, 2025). The waveform data from stations of the Belgian seismic network since 2023 are available in the Orfeus European Integrated Data Archive (EIDA). Older data are scheduled for gradual backfilling in the near future, but can in the mean time be requested from the authors. Python code to compute local magnitude according to different models can be consulted at <https://gitlab-as.oma.be/kris/robspy/-/blob/master/rob/MbLg.py>. The earthquake catalog of the Royal Observatory of Belgium is available at <https://inspire.seismology.be/geoserver/inspire/wfs?service=WFS&request=GetCapabilities>. The earthquake catalog of the Royal Netherlands Meteorological Institute was accessed at <https://rds.knmi.nl/fdsnws/event/1/>. The earthquake catalog of the Bensberg network of the University of Cologne was accessed at <http://www.seismo.uni-koeln.de/data>. Supplemental Material for this article includes seven figures and four tables.

Competing interests

The authors declare that they have no conflict of interests.

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