

Describing the uncertainty of earthquake focal mechanisms

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Abstract Uncertainty estimates on earthquake focal mechanism (FM) data are often simplified to a single, standard error angle or to a qualitative metric because a practical description of the full, 3D rotational uncertainty has not existed. We introduce the rotation vector distribution matrix (RVDM), a 3x3 matrix description of FM uncertainty. The RVDM is a multivariate normal probability distribution of rotation vectors that can easily be computed from a representation of the FM posterior probability distribution (PPD), usually obtained via a grid-search or Monte Carlo method. We investigate RVDM precision in describing synthetic FM PPDs, relying only on P-wave first-motion data, and find that it significantly improves PPD fits compared to simplified metrics. We further examine the impact of propagating the RVDM into synthetic tectonic stress inversions by comparing to inversions that incorporate only a standard (uniform) rotational uncertainty for each FM, as well as inversions that incorporate the full FM PPDs. Finally, we apply our proposed RVDM approach to Monte Carlo PPD samples of FMs, derived from P-wave first-motions, near the Yakutat microplate collision in Yukon, Canada. The results from both synthetic and real data suggest that using the RVDM model (or retaining the full PPD) in tectonic stress inversions can greatly reduce stress tensor uncertainties when the FMs are poorly constrained.

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Non-technical summary Estimating the orientation of faults (planar cracks in the Earth's crust on which earthquakes slip) from the earthquake waveforms recorded on seismometers is an important part of how seismologists interpret the ongoing tectonic deformation in a region, and its seismic hazard. This report introduces a new method to describe the range of possible orientations for an earthquake fault plane (or focal mechanism), which we call a Rotation Vector Distribution Matrix, or RVDM. We perform a series of tests on simulated earthquakes and find that the RVDM model is an effective way to describe focal mechanism ranges, and is an improvement over previously-used methods. We simulate our ability to estimate the orientation of tectonic stress in the Earth's crust from earthquake focal mechanisms, with and without the RVDM model, and find that it is helpful in obtaining a more precise estimate of stress. We end with an application of the RVDM model to earthquakes from southwest Yukon, Canada, where active tectonics are complex and available earthquake data are abundant.

1 Introduction

Quantitative uncertainty estimates on earthquake focal mechanism (FM) data are important if they are to be used for geological interpretation of any kind, most notably for evaluating the tectonic stress tensor and its associated uncertainty. However, describing and adopting the uncertainty associated with earthquake FM estimates can be challenging given the rotational nature of their mathematical representation. Here we outline several of the most common existing approaches, each with their respective advantages and disadvantages.

1. *Uncertainties on individual focal-mechanism components.* In QuakeML (Schorlemmer et al., 2011), perhaps the most common way earthquake data is shared, including via the tools offered in ObsPy (Krischer et al., 2015), there is no class of uncertainty for an FM in its entirety; instead, uncertain-

ties are input on individual components of the FM, e.g., on the strike of first nodal plane or the plunge of the pressure axis. In principle, this allows for a detailed description of uncertainty. However, the errors associated with each FM element are not independent. The uncertainty is thereby described with more degrees of freedom than actually exist and covariance between elements is not accurately characterized. Many studies that use FMs to investigate tectonic stress have adopted such an approach, and assigned uncertainties to each strike, dip, and rake, although they typically assign a single standard error angle to each element (e.g., Balfour et al., 2011; Verdecchia et al., 2022).

2. *Uniform rotation angle.* A more straightforward way to describe FM uncertainty is in terms of rotation of the entire focal sphere. Previous studies reporting rotational uncertainty have generally reported a single angle in degrees, either a standard devi-

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ation or root-mean-square (RMS) rotational error (e.g., Maeda, 1992; Plourde and Nedimović, 2021). This approach is simple, but it does not retain any information on which rotation directions are most likely (i.e., asymmetry in the FM rotational uncertainty).

3. *Error/uncertainty in the moment tensor.* A numerically simple, albeit less physically intuitive, way to describe FM uncertainty is in terms of the FM's unit moment tensor (MT) representation. This could be done via confidence intervals on each MT element, or via an overall “distance” metric between MTs as a dot product $\sum_{i=1}^3 \sum_{j=1}^3 \mathbf{M}_{ij}^a \mathbf{M}_{ij}^b$ (assuming here that $\mathbf{M}_{ij}^a$ and $\mathbf{M}_{ij}^b$ have unit norms) that ranges between 1 and -1 for identical and opposite FMs, respectively. Tape and Tape (2016) introduced the matrix-angle MT confidence metric, computed as the arccosine of that dot product. This provides a robust measure of similarity in the FM radiation pattern, although only as a scalar metric.
4. *Matrix Fisher distribution.* Walsh et al. (2009) present a description of the FM PPD based on a 3D Matrix Fisher distribution. They demonstrate that this model of FM PPD can be described with nine independent parameters, including three that indicate the “mode” or most-probable FM and six that describe the PPD shape (i.e., the uncertainty). These six include three “concentration” parameters that describe the overall uncertainty of each FM axis, and three “spin” parameters that describe how the PPD rotates about each axis. This comprehensive description of the FM PPD is computationally expensive to evaluate. Consequently, Walsh et al. (2009) in practice used a simplified version with a single, scalar concentration parameter that does not allow uncertainty to vary about different rotation axes.
5. *Full probability distribution.* Gosselin et al. (2024) avoided parameterized representations of FM uncertainty by saving the full posterior probability distribution (PPD) of the FMs derived from a Monte Carlo sampling algorithm (Vasyura-Bathke et al., 2020), using P-wave first-motion data. They could then directly sample from the PPD when inverting for the stress tensor with a method that produces many realizations of random noise (Vavryčuk, 2014). This enables accurate propagation of FM uncertainties to stress inversion. However, unlike the previous methods, it does not provide a concise description to be documented and shared, and the approach does not provide an overall uncertainty metric that can be used for FM-weighting in stress inversion.

Beyond FMs, there is also a much broader range of research discussing probability distributions of circular and spherical orientations. These generally build upon the von Mises-Fisher (VMF) distribution for points on the surface of an n -dimensional sphere, or the Bingham distribution for axial distributions in n -dimensions.

These have long been used in the geosciences, in 3D space $\mathbb{R}^3$, to describe distributions of paleomagnetic axes on the regular $\mathbb{S}^2$ sphere (Mardia, 1981). A similar formulation was recently used by Gosselin et al. (2023) to characterize the probability distribution of phase spectra in seismic waveform data. To describe orientation distributions of 3D objects $\text{SO}(3)$, rather than single axes, the VMF and Bingham distributions are commonly extended to $\mathbb{R}^4$, where unit quaternions lie on the $\mathbb{S}^3$ hypersphere (and a unit quaternion describes a 3D rotation or orientation). The Bingham-quaternion method could in principle describe FM distributions effectively, and it has had similar applications in crystallography (e.g., Kunze and Schaebe, 2004; Niezgoda et al., 2016), but, as stated by Kagan (2013) in the context of FMs, the difficult computation of the required normalization factors tends to make the method impractical. Kagan (2013) instead presents a VMF-quaternion description of FM distributions (that does not account for axial symmetry), although not in the context of describing uncertainty of a single FM estimate.

In this paper we present a method to describe rotational uncertainty in detail. The approach is simple to compute so that FM uncertainties can be faithfully propagated to later applications, most notably for inverting a tectonic stress tensor. We test how the use of our approach affects uncertainty in stress inversion from FM data, and compare results with those obtained from uniform rotation angle and full PPD uncertainty methods. We do this for both synthetic data as well as for clusters of FMs for earthquakes near the Yakutat microplate collision with North American, NW Canada (Gosselin et al., 2024).

2 Rotation Vector Distribution Matrix

As an alternative to the previously discussed methods for describing the uncertainty of focal mechanism estimates, we introduce the rotation vector distribution matrix (RVDM). This builds upon the description of rotation angle distributions by Kagan (2013) and Kagan and Jackson (2015) as rotational VMF distributions, underlain by a 3D normal distribution. However, we will forego their use of quaternions and conceptualize the distribution in terms of rotation vectors, a choice examined further in Supplementary Section S1. A rotation vector

$$\mathbf{r}_k = (r_x, r_y, r_z) \quad (1)$$

describes a rotation about the vector, with the vector's length indicating the rotation angle (degrees counter-clockwise, in our convention). The RVDM ($\mathbf{C}_{ij}$) is a 3×3 covariance matrix and describes a multivariate normal probability distribution of rotation vectors, forming an ellipsoid in 3D space (Figure 1). Equivalently, the RVDM describes the probability distribution of a FM disorientation relative to its centroid or best-fit FM, where disorientation refers to the minimum rotation angle between two FMs (often called the Kagan angle; Kagan, 2007, 2013). Note that there are always multiple ways to rotate from one FM to another, including about different axes and by adding any number of 180° incre-

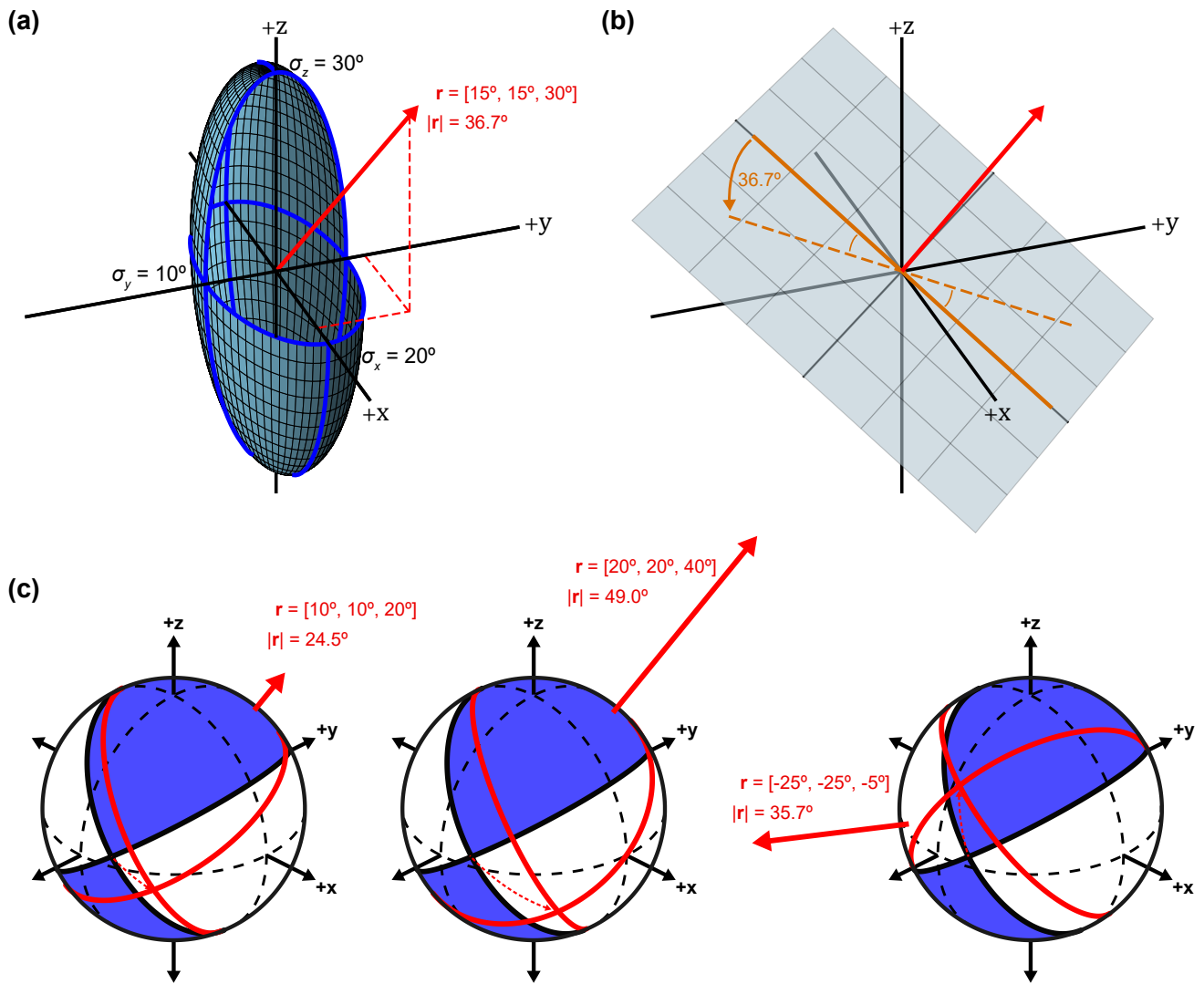


Figure 1 (a) An example, schematic RVDM ellipsoid corresponding to a RVDM and rotation vector. The standard (1σ) ellipsoid for a diagonal RVDM with $[C_{xx}, C_{yy}, C_{zz}] = [400, 100, 900] = [(20^\circ)^2, (10^\circ)^2, (30^\circ)^2]$, and $C_{i \neq j} = 0$. A quadrant of the ellipsoid ($+x, +z$) is cut out to reveal the origin. The red arrow indicates an arbitrary sample from the RVDM, a rotation vector $r = [15^\circ, 15^\circ, 30^\circ]$. (b) The resulting rotation of $|r| = 36.74^\circ$ about the vector $\hat{r}$ illustrated in the plane normal to r , arbitrarily highlighting the axis originally along-dip. (c) Three example rotations applied to an arbitrary FM (360/56/084), using 3D perspectives of the focal sphere. Red lines indicate the rotated nodal planes, dashed black lines mark equators in the x-y, x-z, and y-z planes. The center rotation vector is twice the length of that on the left and corresponds to twice the rotation angle. A distinct rotation vector is shown on the right to illustrate how vector (or axis) direction controls the sense of rotation. The radius of the focal sphere is arbitrary, there is no relation between scale for which it is drawn relative to the rotation vectors.

ments. The disorientation is not the only possible metric to describe the difference of two FMs (there is e.g., the matrix angle, [Tape and Tape, 2012](#)), but disorientation retains an intuitive connection to physical rotation and the rotation-vector construction allows the RVDM to describe distinct rotational uncertainties about different axes.

Knowledge of the probability distribution of an earthquake's FM will generally be achieved from a method that 1) performs a grid-search over the full range of possible FMs, and/or 2) uses a form of bootstrap or Monte Carlo sampling in order to account for uncertainty in the data. For example, [Hardebeck and Shearer \(2002\)](#) do both of these things, as they repeat a grid search over FMs for a variety of perturbed hypocenter and velocity-

model estimates.

From grid-search data with N_{grid} mechanisms, the RVDM can be computed as:

$$\mathbf{C}_{ij} = \sum_k^{N_{\text{grid}}} p_k \mathbf{r}_k \mathbf{r}_k^T, \quad (2)$$

where p_k is the normalized probability of a “grid-point” FM and $\mathbf{r}_k$ is the rotation vector from the mechanism to the best-fit FM, which can be computed using the [Kagan \(2007\)](#) method. If a bootstrap method is applied to compute the FM, N_{grid} can instead refer to the number of bootstrap iterations—in this case all realizations might be assigned equal probability $p_k = N_{\text{grid}}^{-1}$. An overall standard deviation angle σ can be computed as

$$\sigma = \sqrt{\text{trace}(\mathbf{C}_{ij})} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}.$$

An issue with the RVDM model, or any model that builds upon a tailed distribution of disorientations, is that the model has tails that extend infinitely, but the range of possible disorientations between two FMs is bounded. The maximum possible disorientation between any two FMs is 120° , but for any given rotation axis the maximum angle is usually lower—the maximum disorientation is 90° if rotating about the pressure (P), tension (T), or neutral (N) axis. As $\sigma \rightarrow \infty$, the effective distribution becomes uniform; this effect will be further explored in Section 3.

For a uniform, or isotropic, distribution of FM probability, i.e., one with no information on the FM whatsoever, Eq. 2 returns a diagonal $\mathbf{C}_{ij}$ where $C_{xx} = C_{yy} = C_{zz} = \frac{\pi^2}{16} = (45^\circ)^2$, with a corresponding standard error $\sigma_{\text{iso}} = \frac{\pi\sqrt{3}}{4} \approx 77.94^\circ$. The mean disorientation of this uniform distribution from any FM is $\sim 75.16^\circ$, as reported by [Grimmer \(1979\)](#) for orthorhombic symmetry ([Kagan, 2013](#)).

In later sections of this paper we will compare how effectively the RVDM model fits PPDs, with its six degrees of freedom, compared to more constricted models that maintain the same overall σ . The most constricted model, labeled $\mathbf{C}_{ij}^{\text{UNI}}$, has one degree of freedom and can be described by the uniform error angle σ being distributed equally across the x , y , and z axes:

$$C_{xx}^{\text{UNI}} = C_{yy}^{\text{UNI}} = C_{zz}^{\text{UNI}} = \frac{C_{xx} + C_{yy} + C_{zz}}{3}. \quad (3)$$

The second constricted model, labeled $\mathbf{C}_{ij}^{\text{PNT}}$, it is constructed by rotating the RVDM to the P, N, and T axes of the FM and zeroing the covariances (and rotating back to the east, north, and up axes):

$$\begin{aligned} C_{PP}^{\text{PNT}} &= C_{PP}, \\ C_{NN}^{\text{PNT}} &= C_{NN}, \\ C_{TT}^{\text{PNT}} &= C_{TT}, \\ C_{PN}^{\text{PNT}} &= C_{PT}^{\text{PNT}} = C_{NT}^{\text{PNT}} = 0. \end{aligned} \quad (4)$$

This has three degrees of freedom, which are the rotational uncertainties about the P, N, and T axes, and again retains the same overall σ as the full RVDM. For convenience, symbols introduced here as well as later sections are compiled in Table 1.

3 Effect of bounds on the RVDM

Before applying the RVDM model of uncertainty to FM inversion, we first ran a simple numerical test examining how the bounds of possible disorientations affect the model fit. For an initial FM of 030/60/080 (an arbitrary choice) we applied RVDMs with a range of C_{xx} from 1 – 90° , and other C_{ij} fixed at 0. For each C_{xx} we produced 50,000 random rotation vectors $\mathbf{r}_i$, applied the rotations, and computed the resulting disorientation to the original FM with the [Kagan \(2007\)](#) method. We then computed the resulting RVDM with Eq. 2 and extract its σ_x , the total scale of its other diagonal components $\sqrt{\sigma_y^2 + \sigma_z^2}$, as well as the overall σ and scalar kurtosis

of the distribution. Kurtosis measures the prominence of outliers in a distribution, where normal distributions have values ~ 3 , higher values indicate more extreme outliers, and vice versa. The results are shown in Figure 2.

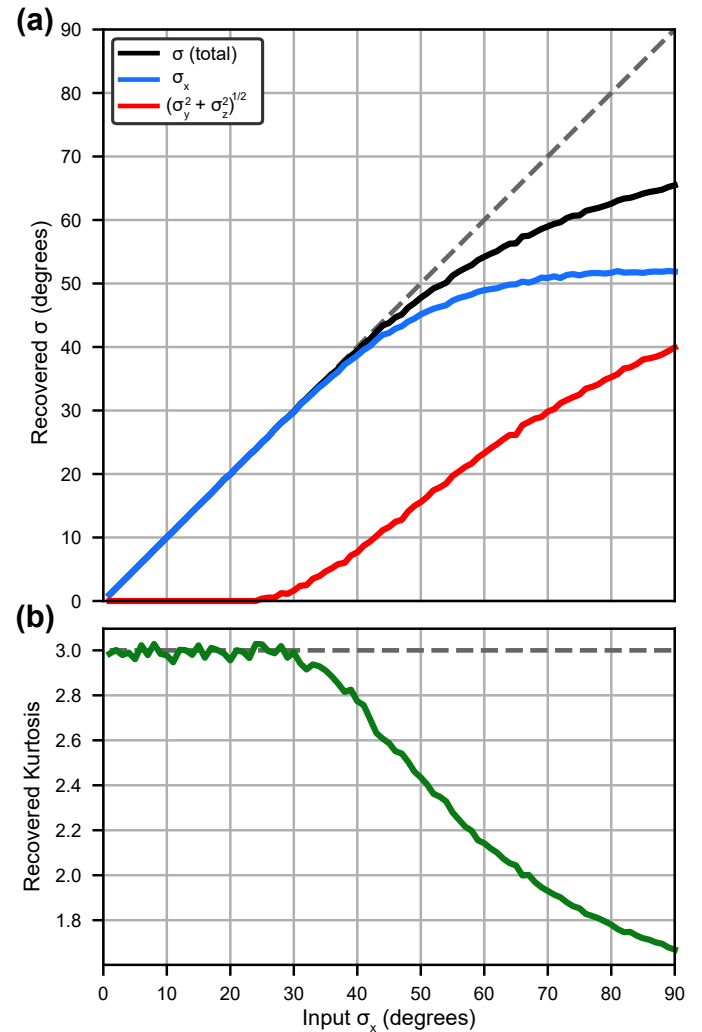


Figure 2 Recovered statistics from 50,000 random rotation vectors as a function of input σ_x , when input $\sigma_y = \sigma_z = 0$. (a) Recovered $\sigma_x = \sqrt{C_{xx}}$, $\sqrt{C_{yy} + C_{zz}}$, and overall $\sigma = \sqrt{C_{xx} + C_{yy} + C_{zz}}$ as a function of input σ_x . (b) Recovered scalar kurtosis, measured as $\sum \frac{1}{N} \|\mathbf{r}\|^4 / \sigma^4$.

For $\sigma_x \lesssim 30^\circ$, the RVDM is accurately recovered as C_{xx} matches the input precisely and other C_{xy} are near zero. For $\sigma_x > 30^\circ$, recovered C_{xx} becomes noticeably lower than the input value, and other $\mathbf{C}_{ij}$ elements become significant. Similarly, kurtosis for $\sigma_x \lesssim 30^\circ$ is always ~ 3.0 , the expected value for a 1D normal distribution, but it begins to decrease as σ surpasses $\sim 30^\circ$. The maximum disorientation about the x -axis (east axis) for this FM is 104.77° , so when $\sigma = 30^\circ$ the bound is at 3.5σ and about 0.05% of the distribution exceeds that bound. At that sigma 1) we begin to notice the effect of bounds on the normal distribution and 2) there begin to be samples that fall in the range $104.77^\circ < \|\mathbf{r}\| < 120^\circ$, where the disorientation axis is not along the x -axis. These disorientations have effectively been mapped to incorrect rotation axes that are not represented in the underlying

Table 1 Summary of symbols used in this study.

Symbol	Definition
$\mathbf{r}_k$	rotation vector
$\mathbf{C}_{ij}$	rotation vector distribution matrix (RVDM) describing FM uncertainty
σ	standard disorientation angle (absolute measure of FM uncertainty) <i>Note the additional uses of σ in different contexts below</i>
σ_{iso}	$\pi\sqrt{3}/4 \approx 77.94^\circ$, the expected σ for an isotropic PPD (information-free case) First appearing in Section 4 (FM inversion tests)
N_{sta}	number of stations (and P-wave first-motion picks) in FM inversion
θ_K	disorientation (Kagan) angle from between recovered and input FM
ϵ	misfit of observed and modeled FM PPD (Eq. 5), sometimes labeled with type of FM-PPD model (UNI, PNT, or RVDM)
$\mathbf{p}_{\text{obs}}$	observed PPD from FM inversion
$\mathbf{p}_c$	modeled FM PPD predicted from an RVDM ($\mathbf{C}_{ij}$)
d_{max}	maximum epicentral distance to stations, in km, for synthetic FM and stress scenarios First appearing in Section 5 (stress inversions)
τ_{ij}	normalized deviatoric stress tensor
τ_5	vector of the 5 τ_{ij} elements we invert for $[\tau_{11}, \tau_{22}, \tau_{12}, \tau_{13}, \tau_{23}]$
s_i	relative principal stresses associated with τ_{ij}
R	stress ratio $(s_1 - s_2)/(s_1 - s_3)$
σ_{eff}	effective σ for weighting stress inversion equations by FM uncertainty (Eq. 6)
ϕ_τ	stress uncertainty metric based on τ_5 (Eq. 7), sometimes labeled with type of FM-uncertainty description (UNI, RVDM, or PPD)
$r_{\text{UNI}}, r_{\text{RVDM}}$	ratios $\phi_\tau^{\text{UNI}}/\phi_\tau^{\text{PPD}}$ and $\phi_\tau^{\text{RVDM}}/\phi_\tau^{\text{PPD}}$
σ_i	orientation of principal stress axes (σ_1 = maximum compression)
u_i	angular uncertainty of σ_i reported at 95% confidence
U	angular measure of stress-tensor uncertainty, calibrated for R (Eq. 9) Specific to Yakutat microplate application (Section 5.2)
θ_g	R -adjusted disorientation angle between stress tensors from this study and Gosselin et al. (2024)
D_g	misfit of our best-fit τ_5 and that of Gosselin et al. (2024), computed similarly to ϕ_τ (Eq. 7)

RVDM. The expected kurtosis of a multivariate normal distribution, with equal scales for each of d dimensions, is $1 + 2/d$, or $5/3$ for a 3D normal RVDM without considering bounds (with $\sigma_x = \sigma_y = \sigma_z$). The expected kurtosis of a 3D, cubic bounded-uniform distribution (3D boxcar) is $19/15$.

This test highlights that the RVDM is a small-angle approximation and will describe the FM PPD most effectively when $\sigma \lesssim 30^\circ$. In the following section, we examine PPD misfits for synthetic FM inversion using P-wave first-motion data.

4 Uncertainty models and focal mechanism inversion

This section examines the use of the RVDM model for synthetic FM PPDs derived from P-wave first-motion data. To compute PPDs from first-motion data, we implement the Bayesian formulation of Pugh et al. (2016) in Python. In addition to a centroid (best-fit) FM, the code returns the PPD across a grid of FMs that fully samples FM-space. The FM grid uniformly samples strike $\in [0^\circ, 360^\circ)$, $\cos(\text{dip}) \in [0, 1]$, and rake $\in [-90^\circ, 90^\circ]$, selecting only right-lateral rakes to avoid unnecessarily sampling both FM nodal planes. We sample strike and rake at $\sim 4^\circ$ increments and $\cos(\text{dip})$ at its equivalent in radians ($\sim 4^\circ \cdot \frac{\pi}{180^\circ}$), resulting in a grid of 66,270 FMs. A best-fit RVDM is computed from the gridded PPD (Eq. 2). The method of Pugh et al. (2016) and our implemen-

tation are described further in Section S2.

4.1 Example P-wave first-motion FM inversion

In order to introduce our FM grid-search method and some of the metrics used to describe FM PPD fits, we first examine inversions of a single FM with three different numbers of first-motion picks ($N_{\text{sta}} = 8, 16$, and 24), selecting a strike-slip input FM for which the cartesian and PNT axes align (strike 45°). The station distribution is randomized and first-motions are given QuakeML phase onset labels, either *impulsive* if its absolute P-wave excitation factor at the focal sphere exceeds 0.25, or *emergent* otherwise; note that this depends only on the ray takeoff direction, station distance is not considered. These labels are then used to assign confidence to each first-motion in the inversion method (Section S2), although for this test no actual noise was added, i.e., the supplied first-motions are always consistent with the input FM. Data for the $N_{\text{sta}} = 8$ and 16 scenarios are subsets of the $N_{\text{sta}} = 24$ to allow for reliable comparison.

Results of this test are presented in Figure 3. In addition to the RVDM ($\mathbf{C}_{ij}$) and its corresponding σ , we also present the disorientation angle θ_K of the computed best-fit FM to the input FM, as well as a misfit metric ϵ_{RVDM} that describes how well the RVDM fits the PPD:

$$\epsilon = 1 - \frac{\|\mathbf{p}_{\text{obs}}\|_2}{\|\mathbf{p}_c\|_2} \cdot \frac{\|\mathbf{p}_c\|_2}{\|\mathbf{p}_c\|_2}, \quad (5)$$

where p_{obs} is the observed PPD from FM inversion and p_{C} the model PPD from the RVDM. We explicitly include normalization terms to set the vector-length of each PPD to 1, such that $\epsilon = 0$ for a perfect fit. In this section we examine fits of the full RVDM (ϵ_{RVDM}), but we will later distinguish it from misfits of more constricted models (ϵ_{UNI} , ϵ_{PNT}).

Figure 3 presents merely one realization of a station distribution and the results can vary significantly depending on how evenly data cover the focal sphere. However, it illustrates some trends that are usually true.

1. Overall uncertainty σ decreases with additional (correct) first-motions more reliably than disorientation θ_K . We also find it very rare to achieve small uncertainties (e.g., $\sim 15^\circ$) with first-motion data even with $N_{\text{sta}} = 36$, at least with the confidence or “noise” properties we assume (Section S2).
2. PPD misfit ϵ_{RVDM} increases with decreasing N_{sta} (with increasing σ), consistent with results from Figure 2.
3. Diagonal components of C_{ij} are usually large compared to off-diagonal components (in this case when the P and T axes align with cartesian axes), suggesting C_{ij}^{PNT} is often a good approximation to C_{ij} .
4. C_{zz} , corresponding to the N-axis, is usually much lower than the other diagonal components. This makes intuitive sense because rotating a FM about its N-axis rapidly changes portions of the focal sphere from positive to negative quadrants, and vice versa, whereas rotating about the P- or T-axis will leave large portions of the focal sphere in their original quadrants. This trend is observed in more broadly in Figure S2, comparing disorientation angle θ_K to the matrix angle metric of Tape and Tape (2016) for the dataset described below.

4.2 Systematic examination of model-PPD fits

In order to investigate how well the RVDM describes FM PPDs, we produced 5,000 synthetic FMs and inverted for them using simulated P-wave first-motions. The number of stations N_{sta} was assigned to either 12, 18, 24, 30, or 36, and the station distributions were randomized for each FM. The station distributions were controlled by 1) randomizing event depths from a uniform distribution between 10 and 20 km, and 2) assigning a maximum epicentral distance d_{max} from which to draw station positions (from a uniform distribution on the 2D Earth surface), set at $d_{\text{max}} = 20, 40, 60, 80$, and 100 km. The distribution in Figure 3 used $d_{\text{max}} = 60$ km.

200 FMs were produced for each combination of N_{sta} and d_{max} , totaling $5 * 5 * 200 = 5,000$. No other controls were added to ensure evenness of focal sphere coverage, so this varied substantially even for a given set of N_{sta} and d_{max} . As with the test in the previous section, for a given FM, the N_{sta} data are always a subset of data of the $N_{\text{sta}}=36$ scenario.

Noise was added to synthetic first-motions in a way that attempts to be self-consistent with assigned noise parameters in the Bayesian formulation but does not rely on phase amplitude measurements. Phases were again given onset labels of *impulsive* for P-wave excitations of >0.25 , or *emergent* for $0-0.25$. Impulsive and emergent picks were labeled as a positive or negative polarity, but with a probability of the label being incorrect that is based on their onset label and is consistent with the noise amplitudes assumed in the FM inversion method (Section S2). Figure S3 compares FM misfits θ_K relative to uncertainties and shows that they behave nearly as expected (there are slightly fewer high misfits than expected), suggesting that first-motions are incorrect at approximately the right frequency to be self-consistent with the assigned uncertainty.

Overall uncertainties as functions of N_{sta} and d_{max} , expressed as a standard deviation angle σ , are shown in Figure 4a. There is a clear trend of decreasing σ with increasing N_{sta} , from median $\sigma \sim 40^\circ$ at $N_{\text{sta}} = 12$ to $\sigma \sim 20^\circ$ at $N_{\text{sta}} = 36$. The relationship with d_{max} is less straightforward, $d_{\text{max}} = 40$ km having the lowest median σ each N_{sta} . It seems reasonable that this middle distance limit provides the best coverage of (upper) focal sphere for earthquake depths of 10 to 20 km, with station takeoff angles concentrated near the equator for higher d_{max} . This observation points to the importance of data coverage (geometry) in accurately characterizing FM uncertainty.

Figure 4b shows ϵ_{RVDM} , ϵ_{UNI} , and ϵ_{PNT} versus σ for the 5,000 synthetic FM inversions. For ϵ_{RVDM} , the 200-FM rolling median is $< 10^{-2}$ for the lowest σ bin (18°), and increases to just over 10^{-1} for the highest σ bin (49°). ϵ_{UNI} and ϵ_{PNT} have similar trends vs. σ as the ϵ_{RVDM} , but are generally much higher. Ratios of $\epsilon_{\text{PNT}}/\epsilon_{\text{RVDM}}$ and $\epsilon_{\text{UNI}}/\epsilon_{\text{RVDM}}$ are shown in Figure 4c to simplify their comparison. Median ratios for 200-FM bins (sorted by σ) are largest for the lowest σ , > 2 for PNT and > 10 for UNI models. Median ratios gradually decrease with increasing σ to 1.1 for PNT and 3.1 for UNI at the highest σ bin (49°).

There are individual cases, for both PNT and UNI models, where the ratio is < 1 , i.e., the constricted model is a better fit to the PPD. This is possible because the misfit metric ϵ_{RVDM} is not precisely what is being optimized when selecting a best-fit FM and RVDM in the FM inversion method. However, in terms of statistical significance, measured by a Brunner-Munzel test, ϵ_{UNI} and ϵ_{PNT} are overwhelmingly greater than ϵ_{RVDM} in every σ bin, with the highest p -value measured at $2 \cdot 10^{-3}$ for the PNT-RVDM comparison at the highest σ bin. Both ϵ_{RVDM} and ϵ_{PNT} are lower than ϵ_{UNI} at p -values of zero to machine precision, for every σ bin.

5 Uncertainty models and tectonic stress inversion

In this section we explore the effects of providing different FM-uncertainty models to tectonic stress inversions, first with synthetic data, then to real examples from earthquakes in Yukon, NW Canada (Gosselin et al.,

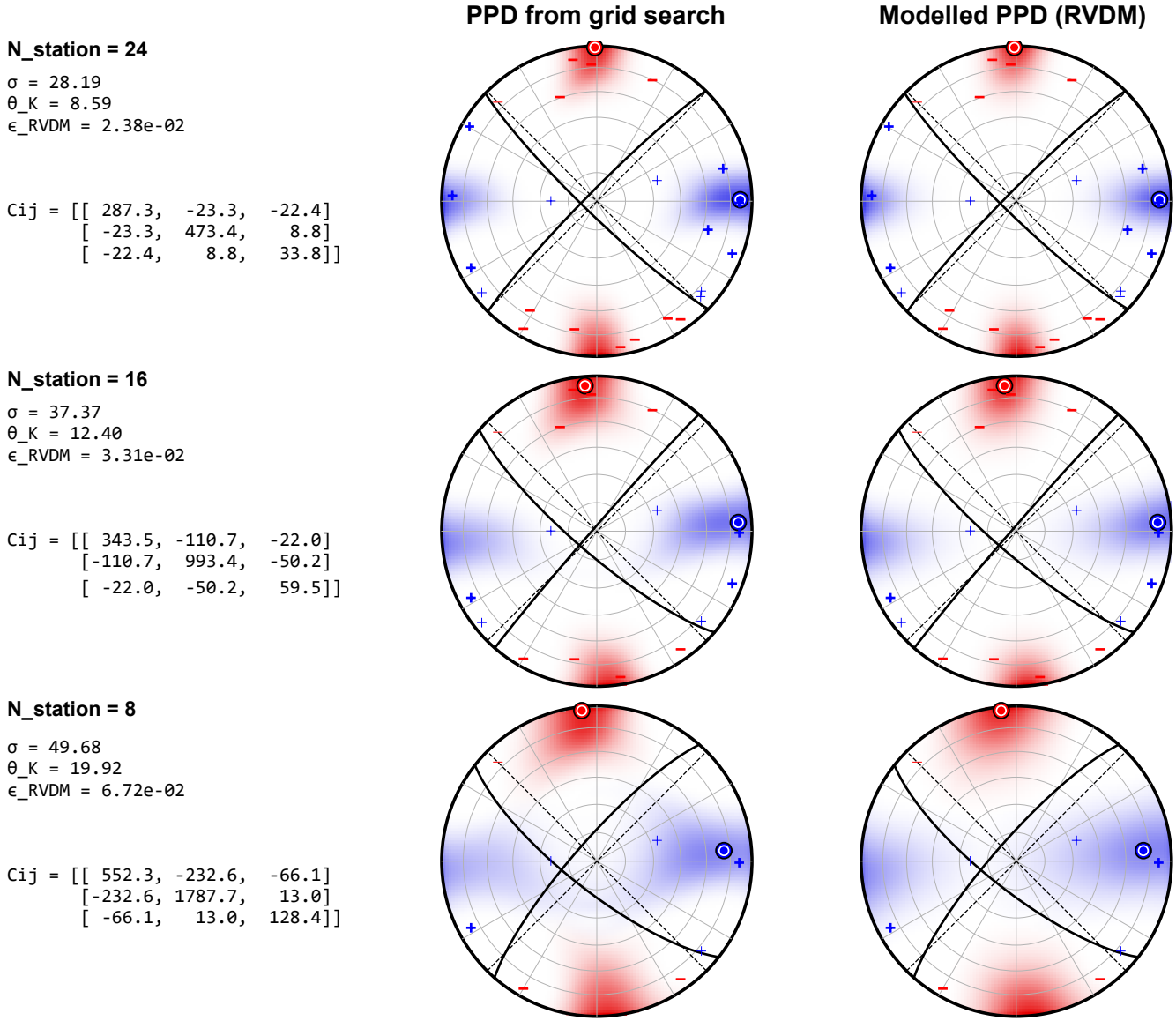


Figure 3 FM inversion results for an input FM with strike, dip, and rake of 045/90/0, such that its principal axes are aligned P = north, T = east, and N = vertical. Three different inversions are shown, with 24, 16, and 8 P-wave first-motions, respectively (top to bottom), where the lower N_{sta} cases are subsets of the 24. All first-motions are consistent with the FM in these preliminary examples, but they are still assigned uncertainty (this contributes to high σ to θ_K ratios). Nodal planes of the best-fit FMs are solid lines, and dashed lines are the input FMs. First-motion data are plotted with blue plus or red minus symbols that are bold if they have high P-wave excitations (impulsive), but not for lower excitations (emergent). Best-fit P and T axes are plotted with red and blue circles, respectively. Blue/red shading illustrates probability distributions of the P/T axes; left panels show computed probabilities from the grid-search, and right panels show the distribution described by the best-fit RVDM. Minor differences between the measured and modeled probabilities can be observed, particularly for $N_{sta} = 8$, for which ϵ_{RVDM} is highest.

2024). Our stress inversion method is implemented in Python, and is described more completely in Section S2, but a few key aspects are highlighted here.

1. The method is built on the linear system of stress, fault normals, and slip vectors formulated by Michael (1984). It outputs a best-fit deviatoric, unit-norm stress tensor τ_{ij} with axes corresponding to principal stresses s_1 , s_2 , and s_3 and stress ratio $R = (s_1 - s_2)/(s_1 - s_3)$, where $R = 1$ indicates uniaxial compression and $R = 0$ indicates uniaxial tension. A vector of the five stress elements in the linearized

inversions $[\tau_{11}, \tau_{22}, \tau_{12}, \tau_{13}, \tau_{23}]$ is denoted τ_5 .

2. It relies on Monte Carlo sampling of the FM PPDs (either the full PPD or as represented by the RVDM) to propagate FM uncertainties to the stress estimate. We use 1,000 Monte Carlo samples for each case in this study.
3. When the nodal plane corresponding to the true fault plane is unknown (which is always the case in this study), we use the Vavryčuk (2014) strategy of performing an iterative set of linearized (Michael,

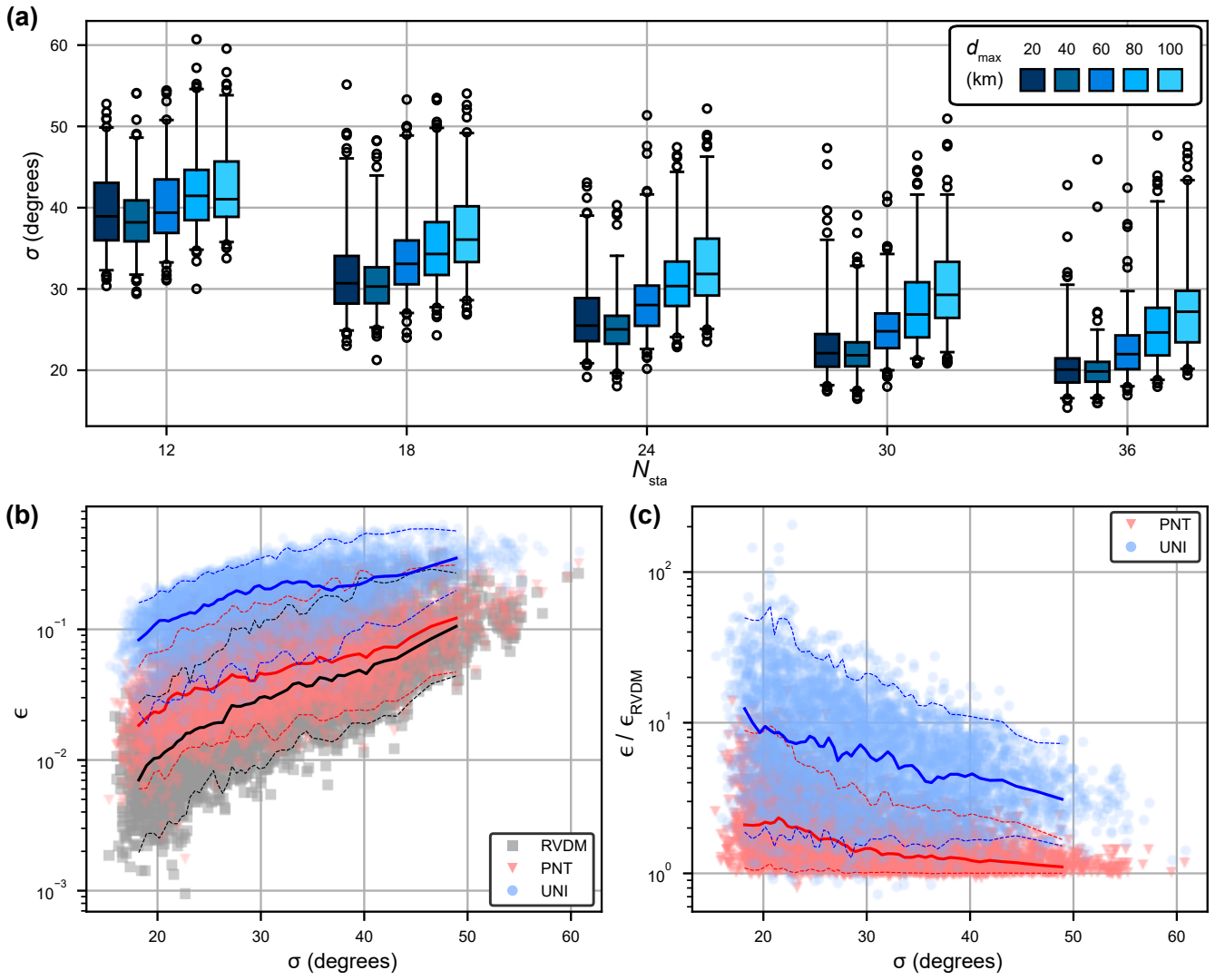


Figure 4 (a) Distributions of standard uncertainty angles (the square-root of RVDM trace or overall variance) for 5,000 synthetic FMs, versus N_{sta} and d_{max} , for each N_{sta} five distributions are shown, corresponding to d_{max} of 20, 40, 60, 80, and 100 km, from left to right. Boxes show the median and interquartile-range (25–75%), with whiskers encompassing the inner 95%. (b) Misfits (ϵ) of the full PPD with that represented by the RVDM, PNT, and uniform models versus σ for each of the 5000 FMs, shown on a logarithmic scale. Solid lines indicate median values for a moving window of 200 FMs, with 50% overlap between windows; dashed lines indicate the corresponding 95% ranges. (c) Ratios of $\epsilon_{PNT}/\epsilon_{RVDM}$ and $\epsilon_{UNI}/\epsilon_{RVDM}$, with solid and dashed lines as in (b).

1984) stress inversions. However, we include both nodal planes in the stress inversion, beginning with them equally weighted. After each iteration, planes are reweighted based on their fit in the current stress estimate, via a combination of the slip-angle and fault-instability methods by default (Section S2.2; Lund and Slunga, 1999; Martínez-Garzón et al., 2016; Chang, 2023) and for real-data applications later in this study. However, for treatment of synthetic data in this study, we will use only the slip-angle method because fault-instability was not considered in randomizing fault planes. This effectively simulates a scenario with extremely high pore-pressure (see Chang, 2023 for discussion on nodal-plane selection).

4. In addition to the nodal-plane weighting scheme, equations in the linear system are weighted rela-

tive to one another according to their FM uncertainties. Rather than using typical σ^{-1} weighting, we account for the bounded nature of rotational uncertainty and weight equations by the inverse of:

$$\sigma_{eff} \propto \frac{\sigma}{\sigma_{iso} - \sigma}, \quad (6)$$

where $\sigma_{iso} \approx 77.94^\circ$ is the expected value for an isotropic PPD, i.e., when there is zero information on the FM. We do not present a formal derivation for Eq. 6, but we note that it satisfies two important criteria: for $\sigma \approx 0 \Rightarrow \sigma_{eff} \propto \sigma$, and as $\sigma \rightarrow \sigma_{iso} \Rightarrow \sigma_{eff} \rightarrow \infty$.

5. Although we compute an RVDM-like matrix for the stress-tensor, with the additional variance of R and its covariance with rotations, the dependence of rotational uncertainty on R makes it more compli-

cated to interpret the overall level of uncertainty from this metric. For example, if $R = 1$, principal stress axes σ_2 and σ_3 become undefined within a plane, meaning there is extreme rotational uncertainty about σ_1 , even though τ_{ij} is well-resolved (note the use of σ_i , for principal stress axis direction, is distinct from previous uses of σ). For this reason, we will mainly report an uncertainty metric based on τ_5 :

$$\phi_\tau = P_{95}(1 - \tau_5^k \cdot \tau_5^{\text{best}}), \quad (7)$$

where τ_5^k indicates a five-element, deviatoric stress vector from Monte Carlo sample k , and P_{95} indicates it is reported at the 95th percentile.

5.1 Synthetic Tests

We first generated synthetic experiments in order to investigate the impact of different FM uncertainty models on stress inversion. These tests comprised combinations of the following elements.

1. 40 input τ_{ij} were produced with random orientation (there were no constraints to ensure one principal axis was vertical), but with R constrained to uniformly cover the range (0,1).
2. 5 sets of 30 synthetic FMs were created per τ_{ij} , created with entirely random fault plane orientations (fault stability in the stress regime was not considered) and τ -consistent slip-directions. For each set of FMs, the 30 hypocenters are randomized around a central epicenter within a 10 km radius, and depths are drawn from a uniform distribution [10, 20] km. 36 station positions were drawn from a disk of uniform distribution with radius $< d_{\text{max}}$, where 5 sets are assigned different d_{max} of 20, 40, 60, 80, and 100 km.
3. 5 subsets of stations were selected from which to invert for FMs using synthetic P-wave first-motions, with N_{sta} of 12, 18, 24, 30, and 36. First-motions were produced as described in Section S2 and, as in the previous section, are noisy.
4. 4 subsets of these FMs were chosen to invert for stress, with N_{FM} of 15, 20, 25, and 30. The subsets are added to sequentially; for example, all 20 FMs from the $N_{\text{FM}} = 20$ set are in the $N_{\text{FM}} = 25$ set.

For each of these $40 * 5 * 5 * 4 = 4000$ scenarios, we performed three stress inversions. Each of these inversions is based on a different representation of FM uncertainties: i) the full PPD, ii) the RVDM, iii) a single rotational variance attributed equally across the x , y , and z axes, with equal overall variance as the RVDM—we again refer to this model as UNI.

Resulting uncertainties on recovered stress for the three FM-uncertainty models ϕ_τ^{UNI} , ϕ_τ^{RVDM} , and ϕ_τ^{PPD} are compared in Figure 5a. Median ϕ_τ decrease with both increasing N_{sta} and increasing N_{FM} ; this trend is most notable for ϕ_τ^{UNI} vs. N_{sta} . At low values of N_{sta} , ϕ_τ^{UNI} is clearly higher on average than ϕ_τ^{RVDM} and ϕ_τ^{PPD} . ϕ_τ^{RVDM} is also higher than ϕ_τ^{PPD} , but by a much smaller margin.

At high N_{sta} , differences between ϕ_τ^{UNI} , ϕ_τ^{RVDM} , and ϕ_τ^{PPD} decrease, but ϕ_τ^{UNI} is always visibly higher.

We assess the statistical significance of differences between the ϕ_τ^{UNI} , ϕ_τ^{RVDM} , and ϕ_τ^{PPD} for a given combination of N_{sta} and N_{FM} with Brunner-Munzel tests; the resulting p -values are listed in Table S1. We find that ϕ_τ^{UNI} is significantly higher than both ϕ_τ^{RVDM} and ϕ_τ^{PPD} for almost all cases, with extremely low p -values at low N_{sta} . At $N_{\text{sta}} = 36$, p -values are near the nominal significance limit of 0.05, and $p > 0.05$ at $N_{\text{FM}} = 25$ and 30. Median ϕ_τ^{RVDM} are always higher than ϕ_τ^{PPD} , but the gap is never significant at a confidence of $p < 0.05$; the lowest p -values occur at $N_{\text{sta}} = 12$.

Figure 6 examines the same stress-inversion results by plotting ratios $r_{\text{RVDM}} = \phi_\tau^{\text{RVDM}} / \phi_\tau^{\text{PPD}}$ and $r_{\text{UNI}} = \phi_\tau^{\text{UNI}} / \phi_\tau^{\text{PPD}}$ vs. ϕ_τ^{PPD} for each scenario, ignoring N_{sta} and N_{FM} . Median r and 95% ranges are computed for 200-inversion bins (sorted by ϕ_τ^{PPD}). Whereas the median r_{RVDM} remains nearly constant at ~ 1.02 , the median r_{UNI} increases from ~ 1.05 at the lowest ϕ_τ^{PPD} to > 1.3 at the highest ϕ_τ^{PPD} . For each bin, we again compared each combination of ϕ_τ^{UNI} , ϕ_τ^{RVDM} , and ϕ_τ^{PPD} with Brunner-Munzel tests and find that ϕ_τ^{UNI} distributions are significantly higher ($p < 0.05$) than the other two distributions in every bin. The ϕ_τ^{RVDM} distributions are also significantly higher than ϕ_τ^{PPD} in 32 of 39 bins, but in general those distributions are much closer.

A second set of box-and-whisker plots in Figure 5b examine stress misfits vs. the true, input stress tensors, defined as:

$$\epsilon_\tau = 1 - \tau_5 \cdot \tau_5^{\text{true}}. \quad (8)$$

We find that ϵ_τ improves substantially with increasing N_{sta} (increasing data quality) but that, unlike with ϕ_τ , only modest improvements are observed with increasing N_{FM} . Furthermore, while $\epsilon_\tau^{\text{RVDM}}$ and $\epsilon_\tau^{\text{PPD}}$ are always lower than $\epsilon_\tau^{\text{UNI}}$, the differences are small and never statistically significant. We believe this is because ϵ_τ depends only on the best-fit τ_{ij} and not its entire PPD. In many inversion schemes, including the Vavryčuk (2014) algorithm, misfit would not depend on the uncertainty estimates at all because the best-fit result is computed directly from the “noise-free” data, rather than from the ensemble of Monte Carlo samples. Our algorithm considers a weighted mean of Monte Carlo results to compute the best-fit τ_{ij} , so the FM-uncertainty model provided can affect ϵ_τ , but we find it affects the uncertainty ϕ_τ much more than the best-fit τ_{ij} and its misfit.

Considered together, the ϕ_τ and ϵ_τ results indicate that using the RVDM model or the PPD of FM-uncertainty reduces ϕ_τ (vs. using the UNI model) primarily because ϕ_τ^{UNI} overestimates stress-uncertainty by providing the inversion FM realizations that are not actually probable and, due to greater propensity of rotations about the N-axis, are more likely to indicate a substantially different stress tensor. This issue is most prominent when the FM PPD misfit ϵ_{UNI} is high, generally at low N_{sta} . Supplementary Section S4 discusses this further and compares the distributions of ϵ_τ and ϕ_τ for each uncertainty model.

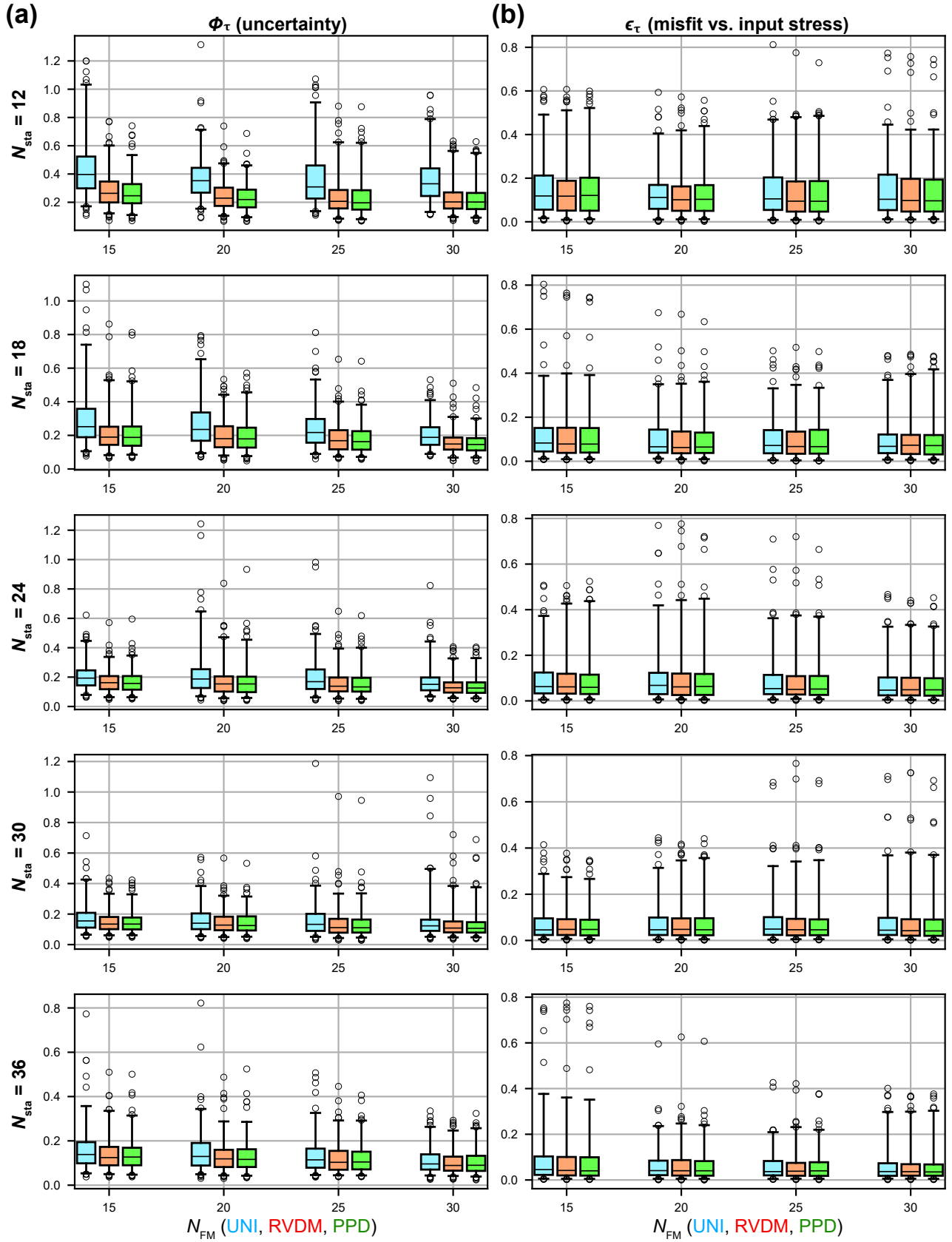


Figure 5 (a) Box-and-whisker plots of stress uncertainties ϕ_τ , where boxes indicate interquartile range and whiskers the inner 95% range. Panels are organized by N_{sta} from lowest (top) to highest (bottom). Within each panel, the x-axis indicates N_{FM} , but for each value of N_{FM} three box-and-whiskers are shown, for ϕ_τ^{UNI} (left, blue), ϕ_τ^{RVDM} (center, red), and ϕ_τ^{PPD} (right, green). (b) Like (a), but for ϵ_τ (misfit) distributions.

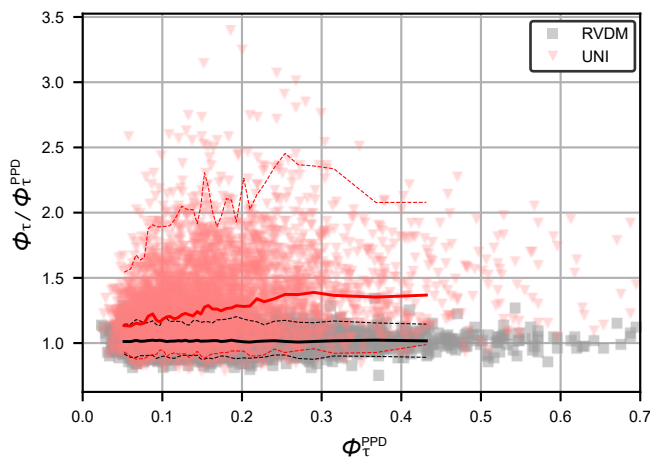


Figure 6 Ratios of stress uncertainties ϕ_θ as computed with simplified FM-uncertainty models vs. those computed with full FM PPDs, r_{RVDM} and r_{UNI} vs. ϕ_{PPD} , for the 4,000 synthetic stress-inversion scenarios. Solid lines indicate median ratios for a moving window of 200 inversions, with 50% overlap between windows; dashed lines indicate the corresponding 95% ranges.

5.2 Application to the Yakutat microplate

As an initial test of the RVDM model to real earthquakes and tectonic stress regimes, we will make use of the Gosselin et al. (2024) catalog of FMs for events near SW Yukon, Canada (Figure 7). SW Yukon sits at the intersection of active plate-boundary deformation and regions of ongoing/expected resource and infrastructure interest (e.g., mineral and geothermal exploration, and major transportation corridors). Paleoseismic and geomorphic studies of the region document multiple Holocene surface-rupturing earthquakes (i.e., $\gtrsim M7$ events), providing longer-term context for the ongoing seismic hazard in SW Yukon (Blais-Stevens et al., 2020; Finley et al., 2025). Regional hazard was underscored by the recent M_W 7.0 earthquake in the St. Elias Mountains of SW Yukon on 6 December 2025 (Finley et al., 2025). This was the largest onshore Canadian earthquake in several decades, which produced significant ground-failure observations (notably landslides/avalanches) despite limited impacts to the built environment due to the remoteness of the epicenter (Figure 7). Because our goal in this work is a methodological validation of FM uncertainty models, rather than event- or sequence-specific reinterpretations, we do not include any FMs associated with the 6 December 2025 earthquake or its aftershocks in our analyses.

Nevertheless, we will briefly review the tectonics of SW Yukon. The region lies within a zone of active deformation associated with the ongoing oblique collision of the Yakutat microplate with North America. Here, the plate boundary transitions from the Fairweather transform fault to the Chugach–St. Elias thrust system, producing a complex pattern of strain partitioning, transpression, and uplift (Enkelmann et al., 2009). Northward motion of the Yakutat microplate, a thick and buoyant oceanic plateau connected to the Pacific plate

(Christeson et al., 2010), is driving convergence and right-lateral strike-slip deformation that is accommodated across a network of major crustal faults (Biegel et al., 2024). Earthquake activity in this region is concentrated in a broad corridor of deformation that characterizes this network of major faults, where stress orientations vary systematically across the plate boundary and into the inboard region, reflecting both plate-boundary geometry and coupling with the subducting Yakutat slab (Eberhart-Phillips et al., 2006; Pavlis et al., 2019; Gosselin et al., 2024). The combination of active deformation, spatially variable stress orientations, and abundant earthquake FMs makes this region a compelling test case for assessing how improved characterization of FM uncertainty influences stress inversion results.

Gosselin et al. (2024) estimated probabilistic FMs for several hundred shallow earthquakes (with magnitudes > 2) using a Bayesian inversion of P-wave first-motion polarity data (Vasyura-Bathke et al., 2020; Gosselin et al., 2022). For each event, the inversion produced a full PPD over strike, dip, and rake values. These probabilistic FMs were then used as input to tectonic stress inversions using the Vavryčuk (2014) method, with a modification to perform resampling of FMs from their respective PPDs to evaluate stress uncertainty. The stress inversions were performed within overlapping bins of 1° in latitude and 2° longitude, chosen as a compromise between spatial resolution and the requirement for a sufficient number of FMs per bin (see Gosselin et al., 2024, for details). They observed a stress regime that is transpressive overall, with a sharp change in the maximum compressive stress direction σ_1 across the Fairweather–Connector–Totschunda fault system (Figure 7). On the eastern side of that fault system, they found that σ_1 rotates from NE–SW in the south ($\sim 60^\circ\text{N}$) to N–S in the northern part of the study region ($\sim 62.5^\circ\text{N}$), such that stress is generally normal to the arcuate active plate boundary.

In the present study, we recompute these stress inversions using each of the UNI, RVDM, and full-PPD FM uncertainty models as in Section 5.1, although in this case our full-PPDs are as represented by 200 Monte Carlo samples (as in Gosselin et al., 2024). We also omit FMs from the International Seismological Centre (ISC) catalog (Lentas et al., 2019) for which we do not have similar measures of their PPDs, but were used by Gosselin et al. (2024). Note that Gosselin et al. (2024) incorporated this additional FM data by estimating uncertainties with a magnitude-uncertainty relationship. This means we have slightly fewer input data than Gosselin et al. (2024), and only 25 spatial bins meet our minimum N_{FM} of 15, vs. 31 in Gosselin et al. (2024). We present results for three example spatial bins in Figure 8 and the complete set of 25 bins in Section S5.

Per spatial bin, Figure 8 reports N_{FM} and the median standard FM uncertainty σ , as well as the diversity of FMs in the bin, defined as the mean-absolute disorientation angle from the mean FM (computed as the best-fit double-couple to the mean MT). A high diversity can indicate a well-posed stress inversion, but diversity can also be artificially increased by high FM uncertainties, if

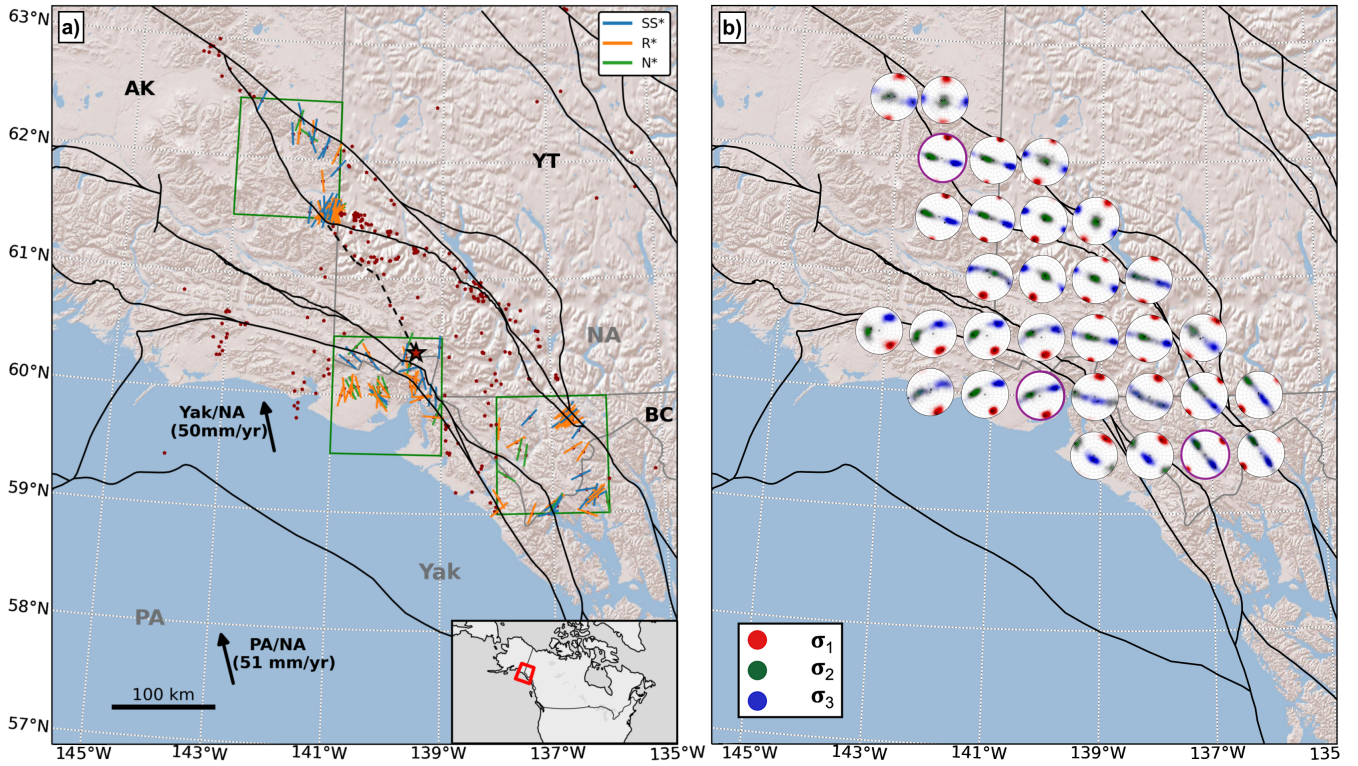


Figure 7 Earthquake FMs and stress orientations near the Yakutat (Yak) microplate and Pacific plate (PA) collision with North America (NA). **(a)** The FM dataset considered by Gosselin et al. (2024). Green boxes denote the three spatial bins considered in Figure 8. Events within the considered spatial bins indicate the orientations of the FM P-axes, colored by the dominant faulting type (strike-slip, SS; reverse, R; normal, N). Black lines show major fault traces and tectonic plate boundaries (Geomantics Yukon, 2018), with the Fairweather-Totschunda Connector fault illustrated by a dashed line. The large red star indicates the epicentral location of the 6 December 2025 M_W 7.0 earthquake. Black arrows indicate plate-motion vectors relative to NA (Altamimi et al., 2023). State, province, and territory abbreviations shown on the map are AK (Alaska), YT (Yukon), and BC (British Columbia). The inset map shows the location of the study area within a broader North American context, with the study region outlined in red. **(b)** Stress inversions from Gosselin et al. (2024). The lower hemisphere plots show the orientation of the principal stress axes for each spatial bin. The red, green, and blue clouds on the lower hemisphere plots represent the distributions of the orientations of σ_1 , σ_2 , and σ_3 , respectively. Specific results discussed in the main text are outlined in magenta.

not properly accounted for. Each inversion is accompanied by its uncertainty metric ϕ_τ (Eq. 7), the plunge and trend of best-fit principal-stress axes σ_i and the best-fit R with its 95% confidence interval, as well as an angular measure of uncertainty:

$$U = Ru_1 + (1 - R)u_3, \quad (9)$$

where u_i is the angular uncertainty of σ_i reported at the 95% confidence level (Plourde and Cassidy, 2022). The metric U provides an intuitive description of uncertainty and does not increase merely because $R \rightarrow 0$ (as σ_1 and σ_2 become undefined) or $R \rightarrow 1$ (as σ_2 and σ_3 become undefined). However, it is sensitive to the best-fit estimate of R , so it is not ideal for examining small differences in uncertainty provided by our three FM-uncertainty models. The differences between our best-fit axes to those from Gosselin et al. (2024) are quantified with both an angular metric θ_g , that is computed analogously to U (Eq. 9) but with a mean R from the two stress tensors, as well as D_g that with the same vector-distance metric as ϕ_τ (Eq. 7).

The three spatial bins in Figure 8 represent cases with

high, medium, and low overall uncertainties (top, center, and bottom rows, respectively). In the first case, with the lowest N_{FM} (24) and highest median σ (52.8°), $\phi_\tau^{UNI} \gg \phi_\tau^{RVD} \gtrsim \phi_\tau^{PPD}$. In the second case with $N_{FM} = 37$ and median $\sigma = 44.8^\circ$, $\phi_\tau^{UNI} > \phi_\tau^{RVD} \gtrsim \phi_\tau^{PPD}$. In the third case with $N_{FM} = 70$ and ϕ_τ that is lowest in general, $\phi_\tau^{UNI} \gtrsim \phi_\tau^{RVD} \gtrsim \phi_\tau^{PPD}$. In the first two spatial bins shown in Figure 8, there are observable differences in our best-fit σ_2 and σ_3 axes, vs. those from Gosselin et al. (2024). However, because R is large in both cases, these do not actually indicate large differences in τ_{ij} , and we see that the differences are low compared to uncertainties in both the vector-dot-product metrics ($D_g < \phi_\tau$) and angular metrics ($\theta_g < U$).

A more comprehensive comparison of ϕ_τ across the three FM uncertainty models is shown in Figure 9a. In agreement with results from synthetic tests in Section 5.1, we observe that ϕ_τ^{RVD} is usually a bit higher than ϕ_τ^{PPD} , and that ϕ_τ^{UNI} is often much higher, but there are exceptions.

Figure 9b examines differences of our best-fit stress tensors and those of Gosselin et al. (2024), using the

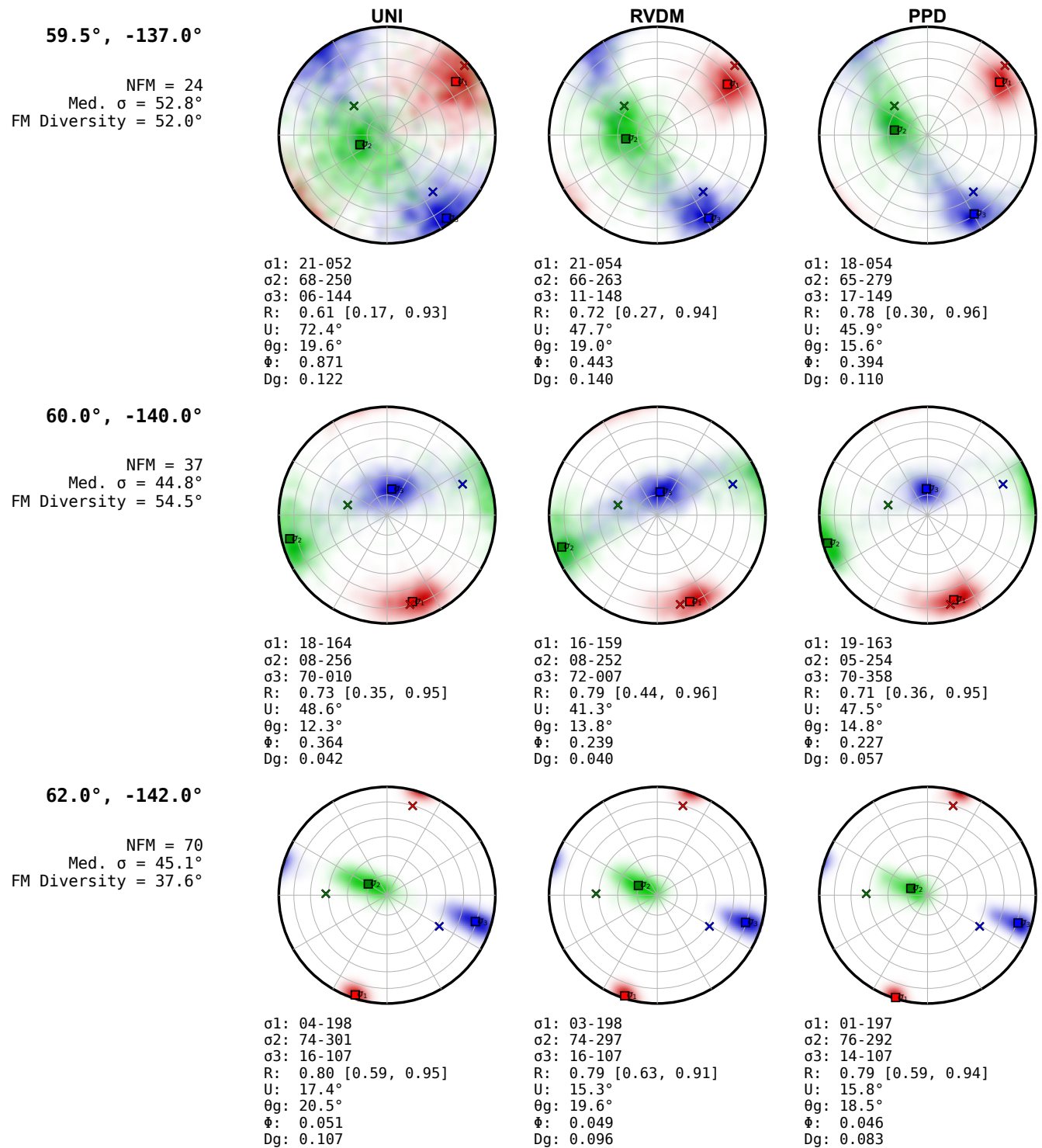


Figure 8 Stress-inversion results for UNI, RVDM, and PPD FM-uncertainty inputs for three spatial bins. Central latitude and longitude, N_{FM} , median overall FM uncertainty σ , and the diversity of FMs for each bin are printed on the left. Note that the symbol σ has two distinct uses in these figures, as it is also used for the principal-axis direction σ_i . Squares indicate the best-fit σ_i axes from this study and “x” symbols mark those computed by Gosselin et al. (2024).

scalar difference metric D_g that can be compared directly with ϕ_τ . We find that in some cases D_g exceeds ϕ_τ , which is reported at a 95% confidence level. From the 25 spatial bins, this occurs for 7 UNI inversions, 5 RVDM inversions, and 6 PPD inversions, but in most cases $D_g < \phi_\tau$. However, we are not accounting for the uncertainty represented by the stress PPD from Gosselin et al. (2024) results in this simplified significance

test (only the best-fit solution is considered), and some differences in stress results are not surprising given the differences between the inversions from Gosselin et al. (2024) and those presented in this work. Most notably, Gosselin et al. (2024) included additional FM data from the ISC, used a different nodal-plane selection metric, and did not consider FM weighting. Figure S5g-i compare simulated stress inversions that consider

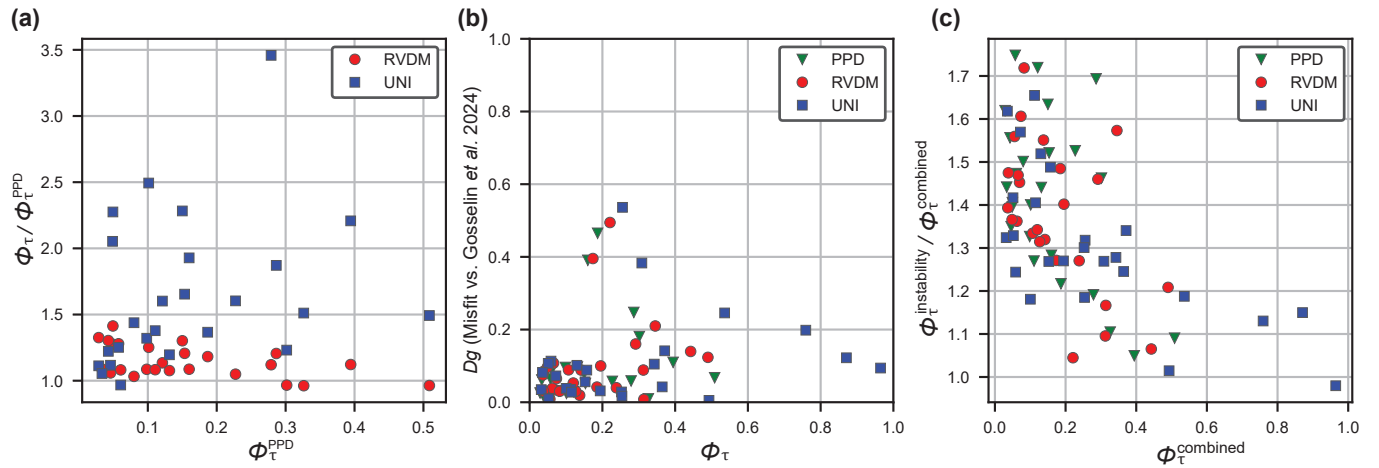


Figure 9 (a) Ratios of stress-tensor uncertainties r_{UNI} and r_{RVDM} vs. ϕ_{τ}^{PPD} (their denominator) for the 25 Yakutat spatial bins. (b) Scalar measurements of stress-tensor differences with the Gosselin et al. (2024) results D_g vs. uncertainties ϕ_{τ} (c) A comparison of stress uncertainties for the two nodal-plane selection methods $\phi_{\tau}^{\text{instability}} / \phi_{\tau}^{\text{combined}}$ vs. $\phi_{\tau}^{\text{combined}}$, for the UNI, RVDM, and PPD FM-uncertainty models. Note that the combined method is used in the other stress-results presented in this section.

FM weighting against inversions that do not, and shows that D between the two cases is generally low but increases with uncertainty. We note that the six anomalous points in Figure 9b at $D_g > 0.3$ correspond to two spatial bins with among the fewest FMs, the largest FM uncertainties, and the largest range of FM uncertainties (see Section S5). The Gosselin et al. (2024) inversions also included many additional ISC events for one of these bins (60.0°N, 137°W), which produced the three $D_g > 0.4$ points. Overall, we do not observe differences in stress results that alter the conclusions of the previous study, and we do not revisit the tectonic implication of the stress inversions here.

Results shown in Figures 8 and 9a treated nodal-plane ambiguity in the stress inversion with a metric that combines the slip-angle and fault-instability methods (Section S2.2). In Section S5, we additionally present a complete set of stress inversions using only the fault-instability metric, similar to Vavryčuk (2014). Although we do not observe any noteworthy differences in the best-fit stress axes across the two methods, we find that the combined metric often produces much lower stress uncertainties. Figure 9c compares ϕ_{τ} for the two cases, for each of the UNI, RVDM, and PPD FM-uncertainty inputs. $\phi_{\tau}^{\text{instability}}$ is higher in all but one case, a UNI inversion with the highest overall $\phi_{\tau}^{\text{combined}}$.

On any given Monte Carlo realization of the FMs, the slip-angle method inherently minimizes misfit in the linear system (i.e., it selects the best-fitting input data) and would therefore always produce lower misfit than the fault-instability method. We note that ϕ_{τ} measures variability across noise realizations, so this result is not similarly obvious, and may indicate that the combined nodal-plane weighting metric leads to true reductions in uncertainty. This could be because for noisy FMs there are cases where the more unstable nodal plane has a slip-direction that actually opposes the overall stress-tensor estimate—an issue alleviated by the combined metric. However, an important difference in the

way the two metrics are implemented is that the fault-instability metric always assigns the full weight (for the FM) to one nodal plane or the other (like in Vavryčuk, 2014), whereas in the implementation of the combined metric we allow nodal planes to share the FM's weight. This difference may effectively make the input data vary more across noise realizations with the fault-instability metric than with the combined metric. Therefore, although we suggest there is good reason to favour the combined metric, we do not consider this to be a statistically robust comparison.

6 Conclusions

Using synthetic P-wave first-motion data to invert for FMs, we found that the RVDM is on average a better fit to the full PPD (ϵ_{RVDM} is low) when overall uncertainty σ is low and wrap-around effects (due to the limit on maximum disorientation between FMs, examined in Section 3) are negligible. Our multivariate Gaussian definition of the RVDM is supported by the central limit theorem for small FM uncertainty. A model built on the Bingham quaternion distribution on $\mathbb{S}^3$ would avoid the wrap-around issues for large FM uncertainty, but we are not aware of a technique that makes normalization of the PPD convenient enough for widespread use as an uncertainty model for earthquake FMs.

With regard to FM-uncertainty propagation to tectonic stress inversion, with both synthetic and real data we found, because the RVDM is a small-angle approximation, that retaining the full FM PPD is most beneficial when σ is high, such that the RVDM has truncation issues and high PPD misfit. This is observed to a small degree in Figure 5a (ϕ_{τ}^{PPD} vs. $\phi_{\tau}^{\text{RVDM}}$), although the improvement we observe in stress uncertainties, versus use of the RVDM, is generally low. In contrast, compared to simpler and more widely used uncertainty models (the UNI model, and we assume other simplified uncertainty metrics), we found that implementing the

RVDM is most useful when FM uncertainties are high ($\sigma > 30^\circ$) and PPD truncation issues are present. This is possible because the simplified models fit the PPD even more poorly ($\epsilon_{\text{UNI}} > \epsilon_{\text{RVDM}}$) at high σ . Taken together, these results indicate RVDM is a useful model for FM uncertainties even at high σ where the small-angle approximation is not strictly valid.

Some may find it more desirable to use C_{ij}^{PNT} given that there are only three values to record and communicate, rather than six, and it tends to fit the full PPD almost as well as the full C_{ij} . However, the full C_{ij} must be computed in order to get to C_{ij}^{PNT} . Consequently, there is little practical benefit its use and we prefer the full RVDM, in part for the convenience of remaining in cartesian coordinates.

Only P-wave first-motion data was tested in this study, so all of the points above should be tested with other forms of FM inversion. Some forms of data, such as absolute-value amplitude ratios (e.g., |P/SV|) and various forms of waveform fitting, may introduce symmetries causing skewed and/or multimodal PPDs. Some sort of clustering algorithm, similar to what Hardebeck and Shearer (2002) and Skoumal et al. (2024) use in HASH, may make it possible to produce a meaningful RVDM from such a PPD, but retaining the full PPD becomes more worthwhile in these cases.

In this study we also presented novel methodology in the process of estimating tectonic stress from FMs, notably including a formulation to weight FMs according to rotational uncertainties that considers the bounds on FM disorientations (Eq. 6), as well as a nodal-plane weighting scheme that considers both fault-instability and slip-angle misfits. However, because we rely on an informal conversion of Monte Carlo stress realizations to stress-tensor PPD instead of a fully Bayesian process, verifying that these measures improve results is not straightforward. This issue highlights the potential benefit of a Bayesian stress-inversion, like the formulation of Arnold and Townend (2007), but the demanding computation required makes such a method a challenge to implement. Arnold and Townend (2007) note that this challenge is particularly severe if trying to incorporate complex PPD models. Consequently, they take advantage of simplifications that arise when considering only a scalar metric of FM uncertainty in order to reduce the complexity.

To conclude, the main purpose of this study was to introduce the RVDM. We have demonstrated that the RVDM model is simple to compute from any FM inversion method that returns a representation of the FM PPD, whether it comes from a grid-search or Monte Carlo sampling, and provides a more faithful description of the PPD than previous simplified metrics. Furthermore, using the RVDM model (or retaining the full PPD) in tectonic stress inversions can greatly reduce stress-tensor uncertainties compared to simpler, widely used uncertainty metrics when the FMs are poorly constrained (low N_{sta} , high σ), with lessening importance as FM uncertainties decrease. Testing how the RVDM model fits PPDs from a wider-variety of FM input-data would be a worthwhile follow-up to this study.

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Data and code availability

Python codes related to FM and stress inversion, including functions to run the synthetic experiments in this paper and FM PPD samples from the Yakutat study, are available on Zenodo (Plourde and Gosselin, 2026). The code builds upon a variety of publicly available Python packages, including ObsPy (Krischer et al., 2015). Seismic data used in the Yakutat FM study are available to download Federation of Digital Seismograph Networks (FDSN) Web Services of the facilities of the Incorporated Research Institutions for Seismology (IRIS; <https://service.iris.edu/>); specific data contributors are described further by Gosselin et al. (2024).

Competing interests

The authors have no competing interests.

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