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Fractal dimensions can lead to spurious interpretations of fault-rock character and slip physics

Randolph T. Williams *[1], **Noah J. Phillips** [2]

[1]United States Bureau of Reclamation, Seismology and Geomorphology Group, Denver, CO, USA, [2]University of Southern California, Department of Earth Sciences, Los Angeles, CA, USA

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Abstract Cataclasis is defined by a combination of distributed microcracking and frictional sliding. These processes lead to mechanical wear and grain-size reduction along fault surfaces during both seismic and aseismic slip. A preponderance of research has utilized "fractal dimensions" extracted from fault-rock particle size distributions as a vehicle to infer the dynamics of strain accommodation and energy release during fault slip. Little work, however, has been done to quantitatively assess the validity of such metrics. Using a series of simple numerical experiments, we show that the fractal dimension provides an unreliable descriptor of fault-rock microstructures and the cataclastic processes that govern their evolution. Our results demonstrate that the most common approaches to estimating fractal dimensions during microstructural analysis are prone to previously unidentified measurement uncertainties. The magnitude of these uncertainties is large relative to what has been considered mechanically and seismologically meaningful by previous work. We conclude that errors inherent in the use of fractal dimensions have likely led to spurious interpretations of fault-rock character and slip physics over the last several decades.

Non-technical summary When faults slip (and potentially produce earthquakes), rocks caught between the opposing sides are subjected to grinding and mechanical wear. This process produces a type of powdered geologic material known as fault gouge or cataclasite. Geologists interpret the physics behind the grinding process by studying the grain sizes of natural and experimentally produced fault gouges. Previously, this process has been interpreted to produce "self-similar" grain size distributions, meaning that fault gouges examined at one magnification look similar to when they are examined at a different magnification (i.e. the distributions are "fractal"). Although recent work has cast doubt on the degree to which these fault rocks are truly fractal, it remains common practice within the geologic community to attempt to estimate the "fractal dimension" of natural and experimental gouge samples as a mechanism of inferring the underlying physics of fault slip. Here, we demonstrate that the common practice of estimating fractal dimensions is subject to substantial uncertainties that were previously unknown. We argue that these uncertainties have likely led to substantial errors in our understanding of the physics of fault slip over the last several decades. We end by proposing several potential pathways toward more reliable characterization of fault-rock particle size distributions.

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1 Introduction

Cataclasis is the primary mechanism shaping the character and properties of fault rocks in the brittle crust (e.g. Sammis et al., 1986, 1987; Marone and Scholz, 1989; Chester et al., 2005). It is defined by the combination of distributed microcracking and frictional sliding, which lead to mechanical wear and grain-size reduction during shear (Engelder, 1974). In this way, fault-rock particle-size distributions (PSDs) are thought to record primary information concerning the micromechanics and energy budgets of slip across the full spectrum of slip behavior (i.e. aseismic creep to dynamic ruptures during large earthquakes). Following pioneering research by Sammis et al. (1986, 1987), it has frequently been assumed that most (if not all) fault-rock PSDs follow a power-law distribution. This assumption allows a fractal dimension "D" to be extracted via linear regression of log-transformed cumulative frequency plots of particle size (Fig. 1). Over the last three decades, dozens of papers have utilized these estimated fractal dimensions as a vehicle to infer: micromechanical models of deformation during granular flow (e.g. Sammis et al., 1986, 1987; Marone and Scholz, 1989; Blenkinsop, 1991; Keulen et al., 2007; Sammis and Ben-Zion, 2008; Melosh et al., 2014); rates and magnitudes of strain associated with fault-rock formation (e.g. Hadizadeh and Johnson, 2003; Hadizadeh et al., 2010; Dyer et al., 2012; Muto et al., 2015; Phillips et al., 2020; Kimura et al., 2022; Berger and Herwegh, 2022; Li et al., 2024); quasi-static vs. dynamic mechanisms of particle fragmentation (e.g. Morgan and Boettcher, 1999; Ben-zion and Sammis, 2003; Melosh et al., 2014; Hadizadeh et al., 2015; Johnson et al., 2021); and systematics of the formation of new mineral-surface area during slip (Chester et al.,

*Corresponding author: rtwilliams@usbr.gov

2005; Dor et al., 2006; Rockwell et al., 2009; Johnson et al., 2021, which consumes some portion of the earthquake energy budget). These and other interpretations, however, can only be considered robust to the extent that the estimated fractal dimension provides an appropriate and reliable measure of fault-rock particle-size data, which has received little quantitative examination.

Two criteria must first be met before the interpretations above (in addition to a wide variety of others) could reasonably gain provisional acceptance. First, the appropriateness of the fractal dimension as a descriptor of cataclastic deformation must be assessed by statistically testing the assumption of underlying power-law scaling in the observed PSDs. Second, linear regression of log-transformed cumulative frequency distributions of particle size must be shown to accurately and reliably extract the fractal dimension from PSDs of known character. Toward the first point, recent work by Phillips and Williams (2021) questioned the underlying assumption of power-law distributed particle-size data in fault rocks by conducting a statistical analysis of a suite of 61 previously published PSDs. Their results showed that best-fit power-law distributions (determined through a maximum likelihood approach) provide a statistically poor fit in each case, particularly when compared to competing log-normal or stretched exponential distributions. Following the publication of Phillips and Williams (2021), however, several recent papers have argued that what our community has frequently referred to as the fractal dimension in fault-rock PSDs (i.e. the slope of a best fit line through log-transformed cumulative frequency data) may still provide a useful metric for describing the characteristics of cataclastic deformation (and comparing those characteristics across previously published works), even if the underlying data do not follow a power-law distribution (e.g. Mancktelow et al., 2022; Kimura et al., 2022; Pizzati et al., 2023; Johnson et al., 2021; Berger and Herwegh, 2022). We accept this possibility, and in this paper we interrogate it further by answering two fundamental questions regarding the validity of fitting a fractal dimension to fault-rock particle-size data, regardless of any assumptions about the underlying distribution:

1. Do common variations in data processing employed between previously published studies affect the fractal dimensions estimated for fault-rock PSDs?

2. Can linear regression reliably extract a fractal dimension from finite samples of a known distribution?

Using numerically generated particle-size distributions, we show that the fractal dimension provides an unreliable descriptor of cataclastic deformation products in most cases. Specifically, we find that variations in data processing between previous studies can result in large discrepancies in the estimated fractal dimension, even when applied to identical data. We also show that the common approach of using linear regression to infer fractal dimensions from cumulative frequency plots of particle size is subject to large errors that are not accounted for by the associated regression statistics, even if the underlying distribution is truly power law in form. We conclude that these errors, which are large and unambiguous, have likely led to spurious interpretations of fault-rock character and physics over the last several decades.

1.1 Community Standards in Fractal Dimension Estimation

The fault and earthquake mechanics communities have utilized several common approaches to estimate fractal dimensions from the PSDs of cataclastic rocks over the last few decades. Most rely on particle diameter measurements estimated in 2-dimensions during microstructural analysis (Fig. 1A,B). These estimates are then converted to a cumulative frequency distribution of particle size before being fitted via linear regression (or in some cases, maximum likelihood) in log-log space (Fig. 1C-E; e.g. Sammis et al., 1987; Marone and Scholz, 1989; Keulen et al., 2007; Melosh et al., 2014; Phillips et al., 2020). The slope of the fitted line is then, in theory, the fractal dimension "D" of the fault-rock PSD (Sammis et al., 1987). A less common approach (as it requires uncohesive samples) is to sort particles through sieving and then to assess relative abundances by weighing each size fraction (e.g. Sammis et al., 1986; An and Sammis, 1994). This study will focus on the image analysis approach, largely given that it is considerably more common with the community and it is also considerably easier to assess via numerical simulations.

There are several matters of practical importance that should be considered when using image analysis combined with linear regression of log-transformed PSDs to estimate fractal dimensions. The first is that plotting and analyzing raw fault-rock particle size measurements necessarily involves some degree of data processing prior to linear regression. As such, the linear regression approach to estimating fractal dimensions, as applied by most of the fault and earthquake mechanics community, is often not being conducted on fault-rock particle-size data per se. Rather, it is being conducted on representations of fault-rock particle-size data, and the nature of those representations varies between previously published studies. These variations are primarily related to the granularity with which binning occurs and/or the granularity with which the number of particles greater than some specific size (i.e. $n_x > x_i$) is evaluated across the particle-size range during the construction of cumulative frequency plots.

Previous work examining fault-rock PSDs has generally employed one of three approaches. We colloquially refer to these approaches as how the data are "binned". One method is to employ a linear binning approach where $n_x > x_i$ is evaluated at a fixed increment of absolute particle size x_i (i.e. every 1 μm) across the observed particle-size range. This approach leads to a heterogeneous distribution of "observations" across the particle-size range in log transformed data (e.g. Fig. 1C). A second binning approach evaluates the number of particles in a bin or $n_x > x_i$ at fixed intervals along the log-transformed particle-size range (e.g.

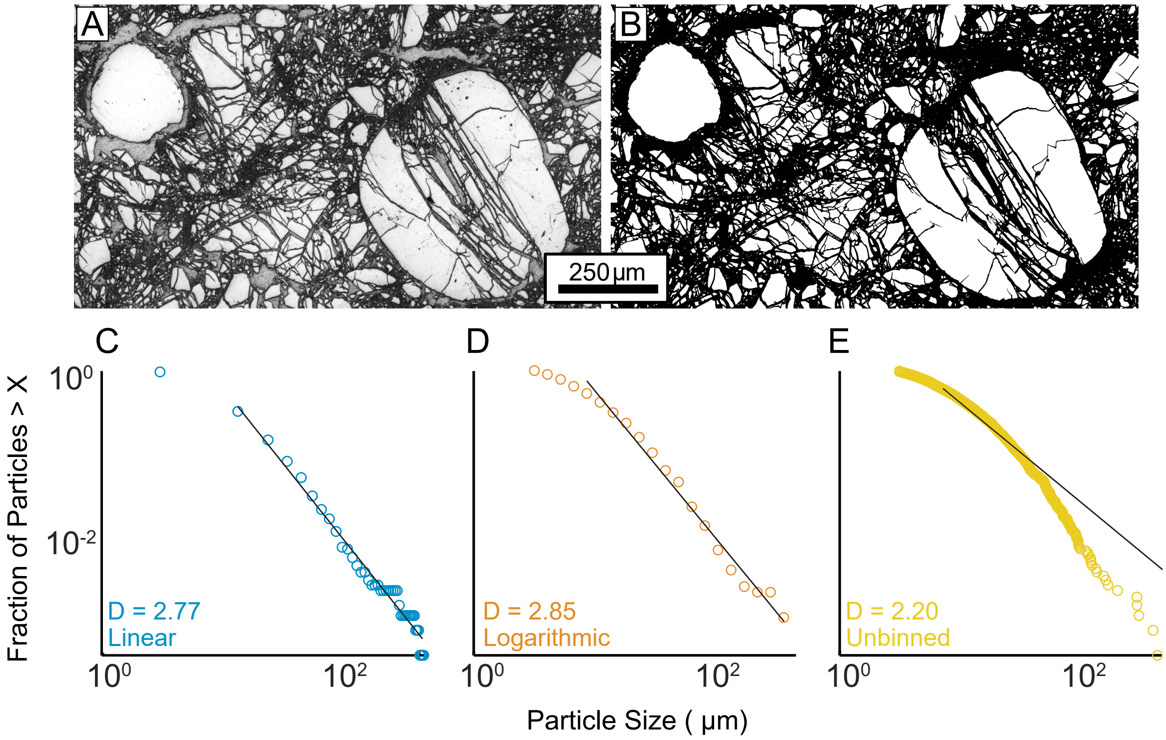

Figure 1 A) Cropped reflected light photomosaic of an experimentally produced quartz gouge (data from experiment GSA04c in Marone and Scholz, 1989). B) Processed binary image of (A) isolating particles for image analysis. C-E) Power law fits to the particles in (B) using linear bins (C), logarithmic bins (D), and unbinned (E) approaches with a minimum grain size of 7 µm. Fractal dimension D = the slope of the linear fit in log space.

Marone and Scholz, 1989; An and Sammis, 1994; Storti et al., 2003; Keulen et al., 2007; Muto et al., 2015; Kimura et al., 2022). This approach leads to a homogeneous distribution of observations across the particle-size range in log transformed data (e.g. Fig. 1D). A final approach is to construct cumulative frequency plots of fault-rock PSDs evaluating $n_x > x_i$ where x_i iterates through each of the actual particle-size measurements or values of x_i (with or without additional conditionals to account for equal-sized particles; e.g. Blenkinsop, 1991; Clauset et al., 2009; Melosh et al., 2014; Hadizadeh et al., 2015; Phillips et al., 2020). We refer to this approach as "unbinned" (Fig. 1E). This approach necessarily biases estimates of fractal dimension toward the bulk of the actual particle-size data. Thus, it is the most data faithful approach that we are aware of, but it will generally result in the application of a fractal dimension to the smallest portion of the observed grain size range, which in most fault-rock PSDs exhibits the greatest departure from linearity and is therefore the least appropriate target for the application of a fractal dimension (Fig. 1E). We note that the bulk of the literature appears to use either logarithmic binning or unbinned approaches. We include the linear binning approach in this study, however, to highlight the effect of binning method on the resulting fractal dimension.

A second issue concerns the specifics of the linear regression analysis that is applied to the cumulative frequency distributions of fault-rock particle-size data to estimate their fractal dimensions. Although much previous work has assumed that fault-rock PSDs follow an underlying power-law distribution, most recognize that PSDs from real fault rocks do not exhibit a true power-law form in practice (i.e. their cumulative frequency distributions exhibit marked departures from log-linearity). Rather, the PSDs generally exhibit a distinct "lower tail" at very fine particle sizes (Figs 1C-E, 2B). The origin of this lower tail is a subject of active debate. Some previous work has interpreted it to be spurious data associated with measurement censoring near the lower limits of microscopy (e.g. Heilbronner and Keulen, 2006). Other work has argued that in many cases the tails occur well above the lower limits of microscopy and exhibit a systematic behavior as a function of shear strain, and thus record a fundamental component of cataclastic deformation (e.g. Sammis and Biegel, 1989; Sammis and Ben-Zion, 2008; Phillips and Williams, 2021; Ord et al., 2022). Regardless, these lower tails, in addition to other departures from linearity in log-transformed cumulative frequency plots of fault-rock PSDs, are the norm. To account for these departures, previous research seeking to quantify fractal dimensions from fault-rock PSDs has opted to employ a minimum particle-size value to restrict the linear regression analysis to only the most log-linear portion of the data set (e.g., a minimum particle size of 7 µm was used in the example shown in Figs 1C-E).

2 How Does Data Processing Affect Estimated D Values In Fault-Rock Particle-Size Distributions?

Given that the linear regression approach to estimating fractal dimensions is often being conducted on representations of fault-rock particle-size data rather than the data themselves, it follows that variations in data processing may lead to disparities in the fractal dimension estimated for any particular fault-rock PSD. In this section, we show that such disparities do indeed arise, and are of sufficient magnitude to impact the legitimacy of comparative analysis of cataclastic microstructures between previously published research.

To evaluate this possibility empirically, we repeatedly draw a finite number of individual particle-size observations (n = 2000; a relatively typical number of individual particle measurements in previous studies) from a synthetic PSD of known character (see associated python code). We then use the standard linear regression approach to estimate a fractal dimension from log-transformed cumulative frequency data with either linear or logarithmic binning. This approach allows us to estimate the potential differences in estimated fractal dimension (as determined via the linear regression approach) that may arise as a function of variations in data processing. For this exercise we employ an underlying log-normal distribution with parameters chosen to be qualitatively similar in form to the raw PSDs observed in real fault rocks (i.e. it incorporates a lower tail at very small particle sizes beyond which it is approximately log linear; e.g. compare natural data in Fig 1C-E with synthetic data Fig 2A; c.f. Blenkinsop, 1991; Keulen et al., 2007; Melosh et al., 2014; Phillips et al., 2020). To be consistent with previous research that has estimated fractal dimensions from real fault-rock PSDs, we employ a minimum particle-size value of 10 µm during linear regression to restrict the analysis to only the most log-linear portion of the data set.

The histograms in Figure 2A show the difference in estimated fractal dimension when the same data are processed using either linear or logarithmic binning in addition to unbinned approaches over the course of 5000 individual trials. The magnitude of the difference is variable depending on the characteristics of the underlying distribution used in the simulation, but in general the results show that differences in data processing alone can lead to discrepancies in the estimated fractal dimension as large as $\sim$0.6 on average. We note that similarly large deviations have been documented to result from variations in bin type when using linear regression in an attempt to extract fractal dimensions in other fields of research as well (e.g. ecology; White et al., 2008).

3 How Reliable Is Linear Regression In Extracting A Fractal Dimension From Finite Samples Of A Known Distribution?

The previous section provides an example of errors that can arise when we attempt to estimate fractal dimensions from fault-rock PSDs using data that were processed differently prior to analysis. An additional issue that can arise as a result of this approach then is that estimated fractal dimensions extracted from the analysis are subject to uncertainties that are not accounted for by the associated regression statistics. Indeed, Clauset et al. (2009) describe in detail several reasons why conducting linear regression on log-transformed data violates many of the underlying assumptions of regression analysis. To evaluate this issue empirically, we employ the same general technique described in the previous section. In this example, however, our goal is to assess the variability/uncertainty in estimated fractal dimensions that can arise for any of the discussed data processing methods (i.e., linear binning, logarithmic binning, or unbinned) when applied to random particle-size observations (n = 2000) from finite samples of a constant distribution of known character. We begin by drawing synthetic PSDs from a true power-law distribution with a prespecified fractal dimension of 2.6 (a value often considered to represent a "mature" stage of cataclastic microstructure; e.g. Sammis et al., 1987). This routine allows us to first assess the reliability of fractal dimension estimation by linear regression in an ideal case. As the underlying distribution is truly power-law in form, we did not employ a minimum particle-size during linear regression of these synthetic PSDs.

In general, the mean estimated fractal dimension in these trials is broadly consistent with the true value of 2.6 in the underlying distribution (Fig. 2C), although estimations produced using linearly-binned data are slightly lower (mean = 2.5) The reliability of the estimates (as inferred from the variability in the estimates following 5000 trials; i.e. the width of the histograms in Fig. 2), however, are more dependent on how the data are binned, once again illustrating the importance of differences in data processing between previous research efforts. When the data are linearly binned, for example, the 95% confidence bound of the estimated fractal dimensions is 1.25–3.34. When the data are logarithmically binned, the 95% confidence bound is 1.84–3.04. These ranges, which we argue must necessarily describe the uncertainty inherent in a single fractal dimension analysis, span nearly the entirety of the range that has been used to describe fault rocks of greatly differing character (either natural or experimental) in previous research (e.g Blenkinsop, 1991; Melosh et al., 2014; Hadizadeh et al., 2015; Muto et al., 2015; Phillips et al., 2020; Marone and Scholz, 1989; An and Sammis, 1994; Storti et al., 2003; Keulen et al., 2007; Kimura et al., 2022; Sammis et al., 1987, 1986; Rawling and Goodwin, 2003). When the data are unbinned, the 95% confidence bound of the estimated fractal dimensions is 2.46–2.78. Although this range is much nar-

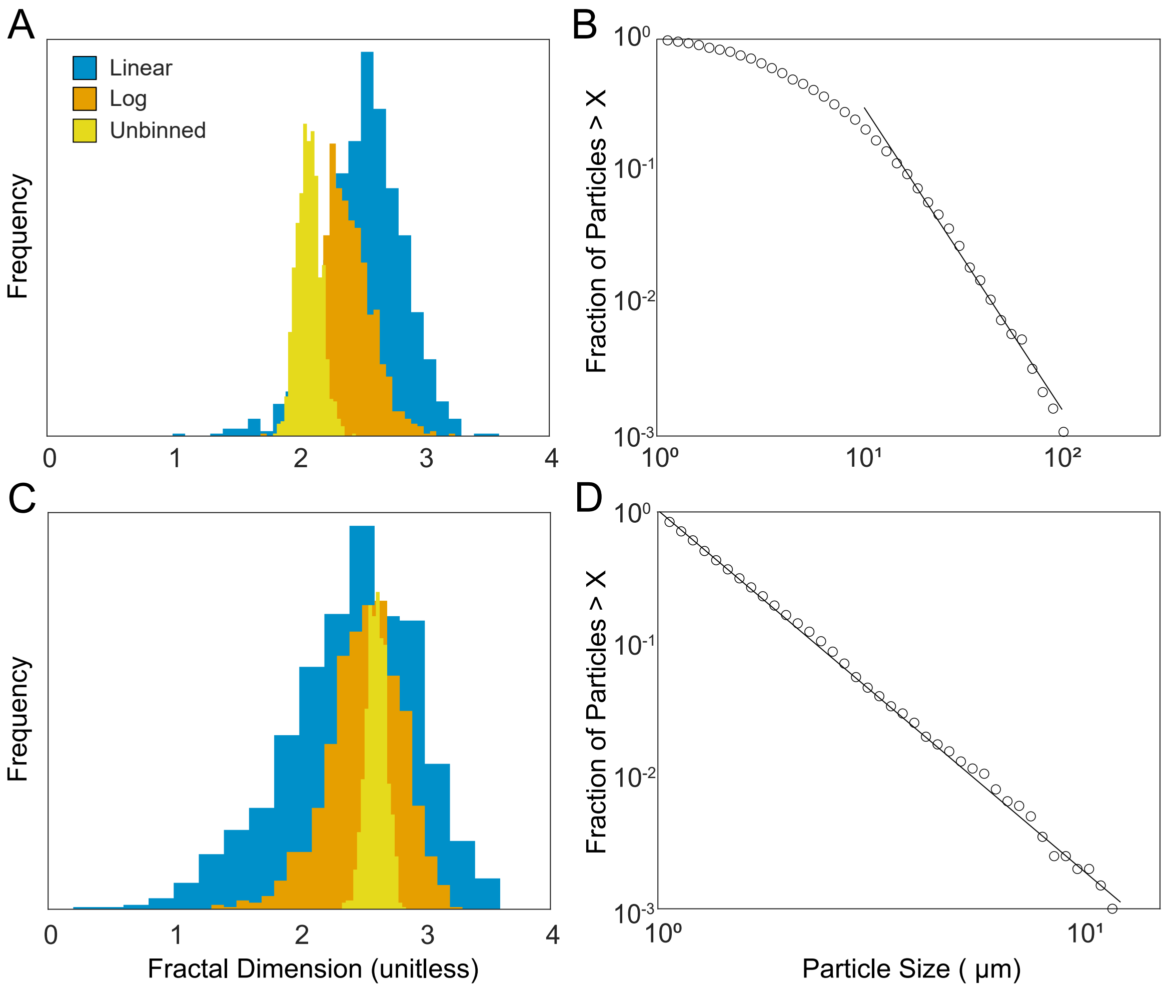

Figure 2 A) Histograms showing the results of fractal dimension estimations determined via linear regression of log transformed cumulative frequency distributions of particle size following 5000 individual trials. Data for each trial was randomly generated from an underlying log-normal distribution parameterized to approximate the form of real fault-rock particle-size data as shown in (B). B) Example synthetic particle-size dataset generated from an underlying log-normal distribution. This example uses logarithmic binning. A minimum particle-size value of 10 μm was used during linear regression. C) Histograms showing the results of fractal dimension estimations determined via linear regression of log transformed cumulative frequency distributions of particle size following 5000 individual trials. Data for each trial was randomly generated from an underlying power-law distribution with a fractal dimension of 2.6 as shown in (C). D) Example synthetic particle-size dataset generated from an underlying power-law distribution. This example uses logarithmic binning. No minimum particle-size value was used during linear regression for the power-law example.

rower than that observed using linear or logarithmic binning, it is still large relative to the between-sample differences inferred to be microstructurally and/or mechanically significant by previous research (e.g. Muto et al., 2015; Johnson et al., 2021; Li et al., 2024). Thus, linear regression of log-transformed cumulative frequency data appears to provide an unreliable estimate of the fractal dimension in fault-rock PSDs, even if the underlying distribution is truly a power law.

As described above, however, real fault-rock PSDs do not exhibit an ideal power-law form (i.e. their cumulative frequency distributions exhibit marked departures from log-linearity). Therefore, and to better assess the reliability of fractal dimension estimates in more realistic samples, we return to the analysis presented in section 2 where we repeatedly draw synthetic PSDs from a log-normal distribution. As discussed in section 2, we choose this particular distribution purely as an example, largely because it is qualitatively similar in form to the raw PSDs observed in real fault rocks. This approach shows that the variability in the estimated fractal dimension (histograms in Fig. 2A) following 5000 individual trials is also large. In comparison to the ideal power-law case described above, the mean estimated fractal dimension across the trials is less consistent between the three binning approaches (as discussed in detail in section 2). As the underlying data are log-normal in form, however, there is no “true” fractal dimension value on which we could determine an absolute error. When the data are linearly binned, the mean estimated fractal dimension for this example is ∼2.52, with a 95% confidence bound of 1.82–3.06. When the data are logarithmically binned, the mean estimated fractal dimension is ∼2.38, with a 95% confidence bound of 2.00–2.81. When the data are unbinned, the mean estimated fractal dimension is 2.08, with a 95% confidence bound of 1.91–2.27. We stress again that in these examples the underlying distribution of the data is identical across all trials.

4 Discussion

4.1 On Linear Regression as a Means of Estimating Fractal Dimensions

The results presented here illustrate several fundamental limitations with the use of linear regression as a mechanism of estimating fractal dimensions for fault-rock PSDs. In section 2, we show that common variations in data processing prior to linear regression preclude the use of fractal dimensions as a meaningful comparator of fault-rock microstructures between previously published studies. For example, both the logarithmic binning and unbinned approaches seem to be employed with approximately equal frequency in the current fault-rock PSD literature (e.g. Melosh et al., 2014; Hadizadeh et al., 2015; Muto et al., 2015; Phillips et al., 2020; Marone and Scholz, 1989; An and Sammis, 1994; Storti et al., 2003; Keulen et al., 2007; Kimura et al., 2022). These previous studies have produced a range of estimated fractal dimensions spanning ∼1–3 for natural and experimental fault rocks, regardless of strain magnitude, lithology, or deformation conditions (either natural or experimental; e.g Blenkinsop, 1991; Melosh et al., 2014; Hadizadeh et al., 2015; Muto et al., 2015; Phillips et al., 2020; Marone and Scholz, 1989; An and Sammis, 1994; Storti et al., 2003; Keulen et al., 2007; Kimura et al., 2022; Sammis et al., 1987, 1986; Rawling and Goodwin, 2003). Between-sample differences as small as ∼0.1–0.2 are regularly interpreted as mechanically and seismologically meaningful (e.g. Muto et al., 2015; Johnson et al., 2021; Li et al., 2024). Our analysis, however, shows that the uncertainties produced by differences in data processing alone may account for discrepancies as large as ∼0.6 when applied to an identical PSD. It is also common practice within the current fault-rock PSD literature to attempt to correlate the estimated fractal dimension of individual samples with those from other studies with better defined temperature/pressure/strain-rate histories. To our knowledge, no previous efforts examining fault-rock PSDs have attempted to control for variations in data processing between distinct data sources. Thus, prior inter-study comparisons are likely subject to uncertainties on the magnitude of those shown in section 2. Critically, those uncertainties span an appreciable fraction of the operative range of estimated fractal dimensions across the aggregate of the fault-rock PSD literature as described above. As such, we argue that many previous efforts leveraging estimated fractal dimensions as a mechanism of extrapolating the characteristics of cataclastic rocks across temperature / pressure / strain-rate gradients documented in the published literature are questionable.

The results presented in section 3 indicate that fractal dimensions determined via linear regression are similarly questionable even as they pertain to describing and interpreting the characteristics of individual samples. For example, the uncertainty around estimated fractal dimensions is rarely reported. Where uncertainties are reported, they are generally derived from the associated regression statistics (e.g. Fagereng et al., 2011). Our analysis, however, shows the range of fractal dimensions estimated via linear regression of 5000 individual datasets randomly drawn from an underlying distribution of constant character is well in excess of those determined via regression statistics. When data are linearly or logarithmically binned into cumulative frequency distributions prior to regression, for example, the resultant fractal dimension range following the 5000 trials again spans nearly the entirety of that used to describe individual fault rocks in the current literature (e.g. 95% confidence bound = 1.3–3.3 when analyzing linearly binned data from a power-law distribution with a fractal dimension of 2.6). The unreliability of the estimate remains even when the analysis is conducted in the ideal case of samples that are drawn from a true power-law distribution. When the analysis is applied to more realistic synthetic PSDs (i.e. those that exhibit lower tails or other departures from log linearity), the uncertainties are slightly smaller (e.g. 95% confidence bound = 1.79–3.1). These smaller uncertainties are of limited utility, however, given that they are still large relative to the range of previously reported fault-rock fractal dimensions, and the discrepancies between different data processing methods become more pronounced when the data deviate from ideal power-law forms (i.e. compare the histograms in Figs 2A,C). It is also the case that our simulations simulated a data set size of 2000 individual particles. Studies utilizing smaller sample sizes (which are relatively common) are likely subject to yet larger uncertainties.

The origin of the issues illustrated in sections 2 and 3 is most readily attributed to the fact that linear regression does not simply fit the "shape" of a 2D data cloud. Rather, the analysis also accounts for the density of observations within the data cloud, which is affected by the choice of data processing approach and the selected minimum particle size for the subsequent regression analysis (Figs 1C-E). An additional issue is the fact that both the linear and logarithmic binning approaches inherently generate estimated fractal dimensions that are explicitly biased against the bulk of the observed data in actual samples. For example, natural and experimental fault rocks are generally typified by a preponderance of very small particles. The nature of the linear and logarithmic binning approaches, however, is such that the majority of the "observations", and thus the results of the linear regression analysis, are skewed toward the largest (and in the original data, rarest) observed particle sizes. As a result, statistical aberrations occurring near the data extremes (i.e. the most sparsely populated portions of the data set, which in this case are the largest particle sizes) likely exert a disproportionate impact on the regression analysis. We note that this same bias is often achieved when some studies employ a minimum particle size for the linear regression analysis in an attempt to fit only the most log-linear portion of the data. These effects are perhaps more clearly visible in our results when comparing trials using linear or logarithmic binning with those employing the more data-faithful, unbinned approach (Fig. 1E).

Collectively, the results presented here raise several troubling questions regarding the validity of past approaches to characterizing cataclastic rocks in fault zones. If we as a community wish to continue using fractal dimensions as a vehicle for describing fault-rock PSDs, then the solution is clear (or at least it is from a purely mathematical perspective): a standardized and more statistically rigorous/repeatable approach to estimating the fractal dimensions is required and raw particle size data needs to be included in data repositories to allow for further analysis. Fortunately, Clauset et al. (2009) provided a detailed methodology (in addition to open source implementations) which can readily achieve that goal. Their methodology employs a maximum-likelihood approach to fitting power-law distributions to empirical data, and thus bypasses the need for linear regression and its limitations in log-log space. The methods documented by Clauset et al. (2009) also include objective approaches to estimating minimum particle-size thresholds and comparing fit quality across competing distributions (with source code). In short, there is a proven and readily accessible method for addressing many of the troubling issues that we have identified in this paper. Our approach to characterizing fault-rock PSDs, and thus any physical or seismological inferences that we deduce from those characterizations, would likely be improved by simply adopting what is already available. It may be more useful, however, if we as a community further consider the possibility that fractal dimensions are simply not as useful for describing the characteristics of fault-rock PSDs as we previously thought.

4.2 On The Utility of Fractal Dimensions as a Metric for Describing Cataclastic Deformation

Whether there is any real utility in estimating fractal dimensions as a means of describing cataclastic fault rocks is largely dependent on the answer to two questions. The first being, are the underlying fault-rock PSDs we are examining truly power-law in form, even if only over a subset of the total PSD range? This question was addressed by Phillips and Williams (2021), who employed Clauset et al. (2009)'s methodology to test for the presence of power-law scaling in a data set of 61 naturally and experimentally deformed fault rocks. Their results showed that none of those data sets exhibited compelling evidence for power-law scaling. Although Phillips and Williams (2021) argued that log-normal and/or stretched exponential distributions provided a significantly better fit to fault-rock PSDs in most test cases, other work has subsequently argued for additional distributions (e.g. Ord et al., 2022). In short, this question has been the subject of several recent quantitative analyses, which have found the evidence for power-law scaling in fault-rock PSDs to be lacking.

A second important question is whether an approximate fractal dimension applied to fault-rock PSD data which are not truly power-law in form still provides a useful descriptor and/or comparator of their microstructural characteristics? If we want our measurements to reliably and uniquely describe the physical qualities of a sample, however, then we suggest that the answer to that question must similarly be "no". The analyses described above illustrate that the common approach to estimating fractal dimensions from fault-rock PSDs does not yield a reliable metric. Fractal dimensions are also a non-unique descriptor of any dataset that departs from an ideal power-law form (as data from virtually all real samples do). When applied to fault-rock PSD data, the application of a fractal dimension merely describes the relative abundance of particles of different size within a specified particle-size range. Any particular fractal dimension determined for a PSD of interest is therefore also likely to describe a large variety of other PSDs, even when the PSDs themselves are demonstrably different in character, as has been previously reported in several studies (Kohlmayer and Grasemann, 2012; Hirose et al., 2012). Perhaps just as importantly, the condensation of an aggregate fault-rock PSD to a single estimated fractal dimension necessarily neglects microstructural characteristics of the sample when the data exhibit any departure from strict log-linearity. For example, Phillips and Williams (2021) showed that many distinct aspects of fault-rock PSD data appear to evolve systematically as a function of total shear strain, including aspects of the PSD that are explicitly excluded during fitting by the application of a minimum particle size (e.g. the "lower tails" of the data set which are removed).

5 Conclusions

The analyses presented here indicate that the common practice of utilizing fractal dimensions as a vehicle for characterizing fault-rock microstructures and the cataclastic processes that govern their evolution is flawed. The errors we highlight above are large and unambiguous, and they have almost certainly led to unreliable interpretations of fault-rock character and slip physics over the last several decades. In light of these observations, we as a community might ask ourselves what exactly we have been measuring for all these years? As of now, it must be said that the answer is not entirely clear, although we recognize some trends of potential value may have been illustrated despite the limitations we present. It does seem likely, however, that we are neglecting important information (or perhaps even forming spurious conclusions) about fault-rock character and the physical processes that shape it by forcing data to conform to our preconceived notions of cataclastic deformation and a desire for expedient analysis.

It is entirely possible (if not likely) that no single statistical distribution or associated metric is capable of fully capturing all the salient features of cataclastic microstructures, which potentially reflect the dynamics of fault slip over timescales from seconds to millions of years throughout the brittle crust. This does not, however, mean that there is nothing to learn about those dynamics from careful examination of fault-rock PSDs. On the contrary, the authors suggest that we are likely only at the beginning of what is possible in that regard. We close by suggesting that a new approach is needed if we are to more fully leverage fault-rock microstructures as a vehicle for examining the physics of fault slip. One possibility is that we could simply interrogate the composite properties of fault-rock PSDs as they appear in cumulative frequency space. After all, collapsing complex data sets to single metrics will almost always neglect information of potential relevance. In that way, there may be no substitute for graphical analysis and semi-quantitative description of fault-rock PSDs and microstructures as a mechanism for capturing the full range of characteristics that can inform the dynamics of slip in the crust.

Data and code availability

Source code for the numerical particle-size distribution experiments can be found at https://osf.io/w2b9h/overview?view_only=f565790933184a3894fec7f3df5c2e8f. Note that this is a private, view-only link.

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Competing interests

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