

Fragility Modeling of Precariously Balanced Rocks: Calibration, Benchmarking, and Sensitivity

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Abstract Precariously balanced rocks (PBRs) provide natural geological indicators for constraining the upper bounds of earthquake ground motions over long timescales. However, translating these constraints into fragility models remains challenging because computationally expensive simulations limit inverse analysis and calibration of contact physics, reducing confidence in the fragility model predictions. To address the challenges, we present a simulated shake-table platform built on a physics engine. Using physical large-scale shake-table experiments on a natural PBR and 582 recorded earthquake displacement histories, we calibrate the contact parameters and benchmark overturning predictions against experimental results and a state-of-the-art discrete element method (DEM). The physics-engine approach reproduces overturning with predictive reliability comparable to DEM, while reducing wall-clock cost by approximately 10^2 to 10^5 times. This efficiency enables large ensemble analyses and allows us to evaluate how uncertainty in contact parameters propagates into inferred fragility boundaries. Among the contact parameters, lateral friction exerted the strongest influence on PBR fragility, whereas restitution and spinning friction had comparatively minor effects, with contact damping and stiffness exhibiting more complex behavior. Our study establishes a practical pathway for using PBRs as quantitative constraints in seismic hazard assessment.

Non-technical summary Precariously balanced rocks (PBRs) are boulders balanced on and not fixed to a sub-horizontal pedestal for thousands of years. Because PBRs have survived past earthquakes, their fragile configurations provide natural evidence of the upper limits of historical ground shaking. This information improves earthquake hazard assessments by helping communities identify sites for critical infrastructures and prepare for future earthquakes. To use PBRs quantitatively, researchers need fragility models that predict the probability that a rock will overturn during earthquake shaking. However, existing approaches based on the discrete element method (DEM) are often computationally expensive and difficult to calibrate. Physics-engine approaches are much faster, but their efficiency and reliability for PBR fragility modeling have not been quantitatively evaluated.

This study addresses two scientific questions. First, how much can a physics-engine approach be trusted for PBR fragility modeling? Second, how strongly does each contact-physics parameter affect the resulting fragility models? We developed a physics-engine-based tool called Virtual Shake Robot 2 (VSR2) to model how these rocks respond to earthquake shaking. We tested the method using laboratory shake-table experiments on a real rock and hundreds of recorded earthquake motions. The results show that VSR2 can predict whether a rock will topple with accuracy comparable to DEM, while running 10^2 to 10^5 times faster. With this efficiency, we are able to quantify how uncertainty in the contact parameters propagates into uncertainty in the inferred fragility boundary, identifying which parameters are weakly versus strongly constrained by single-axis ground motion. Overall, this work provides a practical way to build fragility models for PBRs that can improve earthquake hazard estimates and support risk assessment, infrastructure design, and public safety planning.

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1 Introduction

Precariously balanced rocks (PBRs) serve as natural negative indicators of earthquake shaking intensity, offering constraints on unexceeded ground motions for long return periods (Brune, 1996; Anooshehpour et al., 2004; Rood et al., 2020; McPhillips and Pratt, 2024; Stirling and Pratt, 2026). Their fragile configurations imply that no ground motion sufficiently strong to overturn

them has occurred since their emplacement (Brune et al., 2006; Rood et al., 2022). Even in tectonically active regions such as southern California, numerous studies have shown that many PBRs have remained upright and fragile for thousands of years (Brune et al., 2006; Rood et al., 2022). This long-term fragility has sparked interest in extending local ground-motion histories, which may reveal that historical shaking has been significantly

less intense than previously expected. In practical applications, PBRs have been used to evaluate and test probabilistic seismic hazard analysis (PSHA) models, especially in the context of critical infrastructure such as nuclear power plants, large dams, and nuclear waste repositories, where highly conservative seismic design and preventative actions are required (Anooshehpour et al., 2004; Hanks et al., 2013; Rood et al., 2020).

To use PBRs quantitatively as constraints on ground-motion exceedance, we need fragility models that predict the probability of overturning under a given level of shaking (Purvance et al., 2008). Developing reliable models remains challenging because PBR response is governed by highly nonlinear and complex dynamics (Housner, 1963; Ishiyama, 1982; Shenton III and Jones, 1991; Zulli et al., 2012; Chatzis and Smyth, 2012). Analytical models have provided foundational insight into rocking behavior but typically simplify PBRs as 2D rectangular inverted pendulums (Housner, 1963). 3D numerical simulations based on the discrete element method (DEM) can represent complex geometry and contact behavior more realistically, although these approaches are often computationally demanding (Purvance et al., 2012; Saifullah and Wittich, 2019, 2022, 2023, 2024). Physics-engine-based simulation offered substantially greater computational efficiency and showed qualitatively good agreement with analytical solutions and desktop shake-table experiments using 3D-printed PBRs (Chen et al., 2024a). Despite these encouraging results, the suitability of the physics-engine approach for PBR fragility modeling remains insufficiently understood. In particular, two key questions remain unresolved: how reliable are physics-engine-based fragility predictions, and how strongly do individual contact-physics parameters influence overturning response?

Addressing these questions requires not only testing predictive accuracy against physical experiments, but also determining whether the computational efficiency is sufficient to support large-scale inverse analysis and calibration. In this study, we present a physics-engine-based approach for modeling the overturning dynamics of PBRs. Using a dataset from prior large-scale shake-table experiments on a natural PBR subjected to 582 recorded earthquake displacement histories (Saifullah and Wittich, 2023), we calibrate VSR2 contact parameters and benchmark overturning predictions against experimental results and a state-of-the-art DEM approach. We also evaluate the computational efficiency of simulating overturning for both a complex-geometry PBR and a prismatic block. Our results show that the physics-engine-based approach can reproduce overturning outcomes with reliability comparable to DEM while providing major computational advantages. These results improve our understanding of the reliability and limitations of physics-engine-based simulation for PBR overturning. Additionally, we use this platform to investigate how individual contact-physics parameters influence overturning behavior, thereby clarifying the role of contact physics under single-axis shaking.

2 Related Work

The dynamics of PBR response to ground motions are complex, as PBRs are naturally occurring, freestanding structures, unrestrained at the base and capable of sliding, twisting, rocking, or overturning. The simplest analytical representation for these freestanding structures is an inverted pendulum model by Housner (1963), which reduces the freestanding structure to a 2D projection of a symmetric rectangular block rocking about two corner contact points. Assuming sufficiently high friction, the motion response is treated as pure rocking without sliding. Even in this basic form, the overturning response is highly nonlinear. Over time, many extensions have been developed, including models with flexible foundations (Psycharis and Jennings, 1983; Chatzis and Smyth, 2012), multibody or multi-block rocking systems (Psycharis, 1990), and formulations that incorporate additional motion modes such as sliding, rocking, translational jumps, and rotational jumps (Ishiyama, 1982; Shenton III and Jones, 1991). Purvance et al. (2008) compared Housner's 2D model with physical shake-table experiments and proposed a logistic regression approach to a probabilistic fragility model. Analytical 3D extensions have also been developed. For example, Zulli et al. (2012) demonstrated that because of the 3D geometry of the model and the response modes involved, rocking can occur at a smaller amplitude compared to the 2D counterpart. Therefore, caution should be practiced while using simplified 2D models. This was further emphasized by Chatzis and Smyth (2012) that 2D modeling cannot adequately represent 3D dynamics. However, incorporating the full 3D geometry and behavior of PBRs into analytical formulations increases their nonlinearity and complexity, making them considerably more challenging to solve.

Numerical approaches offer an attractive alternative to analytical formulations. Veeraraghavan et al. (2016) analyzed 3D PBR overturning dynamics using a constraint-based model that formulates a linear complementarity problem from contact constraints and solves for contact impulses iteratively to update object velocity. Their results showed strong agreement with the shake-table experiments of Purvance et al. (2008). A general numerical approach is the DEM, which simulates rigid blocks with deformable contacts and solves the equations of motion using explicit time-integration schemes. Purvance et al. (2012) used DEM to analyze the overturning response of 3D PBR models. Saifullah and Wittich (2019, 2021, 2024) employed DEM to model seismic response of freestanding structures. Focusing on PBRs, Saifullah and Wittich (2022, 2023) used DEM to evaluate the overturning uncertainty from rock interface parameters, material, and ground-motion parameters. Their studies conducted real shake-table experiments on a natural PBR, providing a valuable experimental benchmark for evaluating the performance of DEM simulations. While representing the state of the art in modeling the overturning dynamics of PBRs, DEM is computationally demanding, with performance slowing considerably for complex 3D geometries and large contact stiffness (Saifullah and Wittich, 2023).

Our previous study introduced the Virtual Shake Robot (VSR), a robotics-based simulation platform that utilizes the Bullet physics engine (Coumans, 2015) to investigate the overturning and post-overturning (large-displacement) dynamics of PBRs (Chen et al., 2024a). While the VSR demonstrated notable progress in simulating PBR dynamics, several key limitations remain. First, assessment using 3D-printed PBRs (Chen et al., 2024b) does not fully capture the physical realism of natural rock behavior. Critical properties such as mass, moment of inertia, and restitution cannot be accurately replicated with 3D-printed models, yet these parameters play a significant role in governing overturning dynamics. Second, the VSR was implemented within the Gazebo simulation toolbox (Koenig and Howard, 2004), which provides a convenient and accessible platform for testing dynamics. However, its limited numerical precision, resulting from reduced bit depth in floating-point representations, restricts the accurate calibration of contact physics. While this level of precision is generally sufficient for qualitative studies and system prototyping, Gazebo does not offer the level of accuracy required for rigorous quantitative evaluation of overturning dynamics, highlighting the need to adopt more advanced simulation platforms for improved numerical accuracy. Third, the previous study did not examine how individual physics parameters influence overturning behavior. Understanding the role of each parameter is essential for uncovering the physical mechanisms of PBR fragility and for assessing the sensitivity of the optimized parameters to the resulting overturning behavior. Giordano et al. (2025) investigated physics engine based simulation for analyzing the rocking behavior of rigid masonry blocks. However, because rocking is a precursor to, but not equivalent to, overturning (Ishiyama, 1982; Yim et al., 1980), physical overturning experiments are still needed to calibrate and validate these physics-engine simulations.

3 Physics-Engine Simulation Platform

We develop a simulated shake-table platform, named Virtual Shake Robot 2 (VSR2), that integrates PyBullet physics engine (Coumans and Bai, 2016) with Robot Operation System 2 (ROS2; Macenski et al., 2022). PyBullet performs the core physics simulation, handling rigid-body dynamics, contact interactions, and motion trajectories. ROS2 manages communication, control, and experiment coordination. Specifically, the VSR2 platform integrates two custom programs: Simulation Node and Control Node, which operate together to simulate the dynamic response of PBRs subjected to horizontal, single-axis ground motions, as illustrated in Fig. 1. The Simulation Node constructs and manages the virtual environment using the PyBullet physics engine. This environment includes a pedestal-actuated shaking platform and a user-defined PBR model with configurable geometry and physical properties. The Control Node generates and executes ground-motion inputs. These inputs are transmitted through ROS2 interfaces as trajectory commands. During each simulation trial, the VSR2 platform records both the commanded and actual motions

of the pedestal, along with the translational and rotational response of the PBR. Fig. 2 records rock response to a real displacement seismogram (from Center for Engineering Strong Motion Data, 2015; Ancheta et al., 2014) simulated in VSR2. This architecture enables detailed analysis of sliding, rocking, and overturning behavior. The modular and reproducible design offers a high-fidelity platform for evaluating PBR fragility under diverse seismic inputs. See the Supplementary Material S1 for full implementation details of the physics-engine simulation platform.

VSR2 supports flexible specification of pedestal and rock properties using human-readable YAML configuration files, enabling systematic variation of parameters without modifying code. Pedestals can be defined as idealized prisms or high-resolution triangular meshes from 3D topographic data. Rocks are modeled as rigid bodies and can be defined as either simple boxes or watertight mesh models derived from lidar or photogrammetry. All physical properties and contact parameters of the pedestal and rocks, including mass, inertia, friction, restitution, stiffness, and damping (Coumans and Bai, 2023), are configured through configuration files, enabling consistent and reproducible simulation setups across a wide range of experiments.

VSR2 simulates pedestal motion through four configurable modes: single cosine, grid cosine, single earthquake, and batch earthquake displacements. The single cosine mode generates a smooth, analytic displacement pulse defined by user-specified amplitude and frequency. The grid cosine mode extends this by systematically varying ground-motion parameters across a grid of peak ground acceleration (PGA) and peak ground velocity (PGV)-to-PGA ratios, two widely used intensity measures in PBR fragility analysis (Purvanche et al., 2008). The earthquake-based modes apply real-world displacement seismograms, either individually or sequentially. In either case, the system resets the PBR between trials to ensure consistent initial conditions. A closed-loop feedback controller samples the reference trajectory at 1 kHz and applies it to the prismatic joint that drives horizontal shaking. Supplementary Material S1.3 shows that this control method enables the pedestal to track the prescribed ground-motion trajectory with high accuracy and precision.

3.1 Simulation Experiment Setup

We designed the experiments to calibrate VSR2, evaluate its overturning predictions against physical shake-table and DEM results, and examine how contact-physics parameters influence PBR overturning behavior. The benchmark data used in our experiments were obtained from Saifullah and Wittich (2023). Specifically, the data that we used included a detailed PBR surface mesh (named “Specimen 1” in Saifullah and Wittich, 2023), real-world earthquake recordings, their DEM simulation results, and corresponding shake-table measurements. The detailed PBR mesh was derived from FARO Focus X130 lidar and reconstructed using the Poisson Surface Reconstruction method (Cignoni et al., 2008; Kazhdan et al., 2006). The earthquake

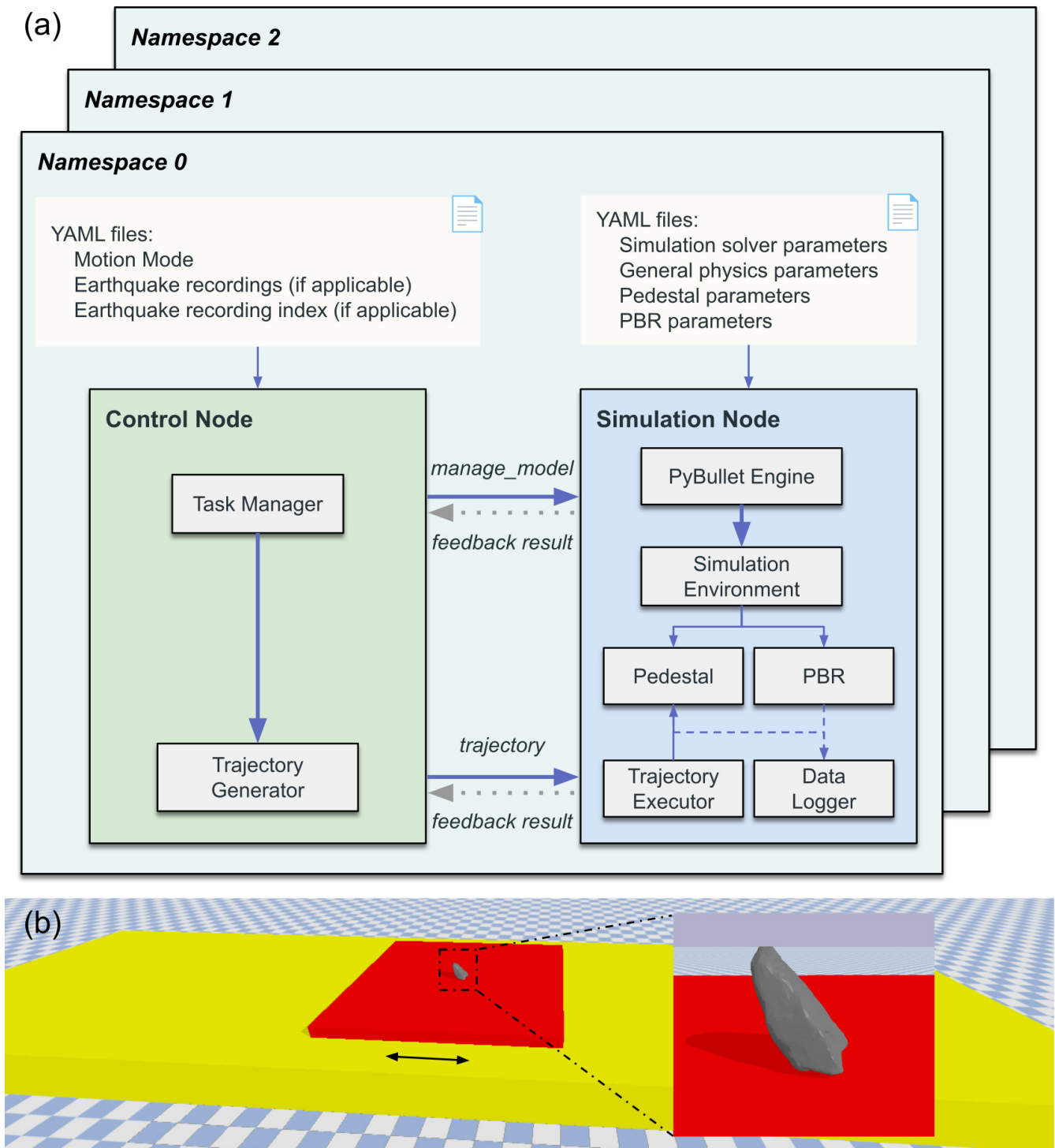


Figure 1 (a) System architecture of the Virtual Shake Robot 2 (VSR2). Each namespace operates an independent simulation instance composed of a virtual shake table with two Robot Operation Systems 2 (ROS2) programs: Control Node and Simulation Node. The Control Node parses motion commands and manages experiment execution, while the Simulation Node instantiates the physical environment using PyBullet and records PBR responses. Human-readable YAML configuration files define all simulation parameters, including ground motions and physical properties. Nodes communicate via ROS2 interfaces for spawning and reset PBRs, trajectory control, and result feedback. Parallel, independent simulations are enabled through multiple namespace instantiations (Namespace 0, 1, 2, ...). (b) VSR2 simulation environment showing the fixed base (yellow), shaking pedestal (red), and the mesh PBR model (gray). A prismatic joint actuates the pedestal movement on the base, with the arrow indicating the motion direction.

recording dataset comprised 582 ground motions, primarily selected from the NGA-West2 dataset (Ancheta et al., 2014) and Center for Engineering Strong Motion Data (Center for Engineering Strong Motion Data, 2015).

These motions span a broad range of intensity measures, characterized by PGA and the PGV/PGA ratio, and were scaled to represent varying levels of seismic demand for PBR overturning analysis. We imported the

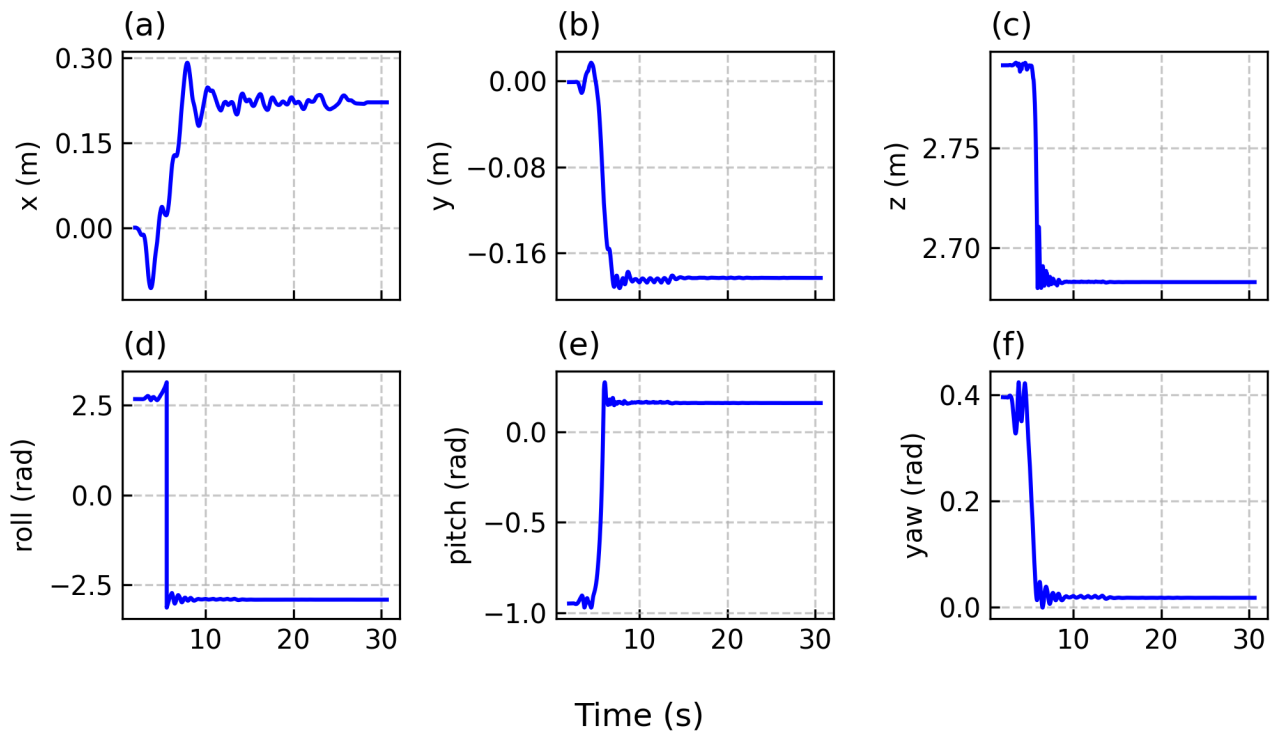


Figure 2 Rock response recorded by VSR2 showing a topple event under real earthquake ground motion applied along the x-axis. (a–c) Position of the PBR’s center of mass over time. (d–f) Orientation of the PBR represented in Euler angles. The coordinates are defined in the simulation’s world frame, where x denotes the ground-motion (shake-table) direction and z represents the vertical axis.

PBR mesh model in VSR2 and conducted overturning experiments using all 582 ground motions under the all earthquake displacement recordings motion mode. Because an objective of this study was calibration of contact parameters and direct benchmarking against the prior DEM workflow (Saifullah and Wittich, 2023), we did not partition the dataset into separate calibration and validation subsets, maintaining consistency with that prior study. Fig. 3 illustrates our experiment setup. The DEM implementation and parameter set used for comparison are described in Section 3.4.

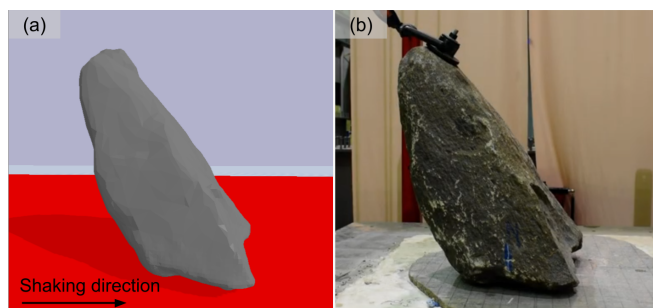


Figure 3 Experimental setup. (a) The VSR2 simulation environment showing the mesh model of the precariously balanced rock (PBR) positioned on a digital pedestal in a virtual scene. (b) The corresponding physical experiment conducted on a large-scale shake table, featuring the natural PBR mounted on a concrete pedestal. The PBR had a footprint of approximately 0.3 m by 0.6 m and a height of 0.8 m, with a mass of 353.8 kg.

3.2 VSR2 Contact Physics Parameters

Our experiments investigated the contact physics parameters in PyBullet, which govern the interaction between the rock and pedestal and control the rock response to ground motions. We focused on five parameters primarily affecting the rock response to single-axis ground motions: restitution, lateral friction, spinning friction, contact damping, and contact stiffness. Table 1 includes the description of these parameters. Other important parameters are the PBR geometry, mass, and moment of inertia. Following the same configuration in Saifullah and Wittich (2023), our experiments used the density of granite 2700 kg/m^3 . Based on the density and mesh geometry, we calculated mass and moment of inertia. In all VSR2 simulations, the PBR and pedestal were assigned the same body-level PyBullet contact parameter values. However, PyBullet resolves contact interactions using effective pairwise contact values computed from the two contacting bodies. The default PyBullet contact-parameter combination rules and the resulting effective pair values are described in Supplementary Material S2.

Supplementary Material S2 also describes contact modeling in PyBullet and explains how object states are updated when impacts occur. In contrast to the DEM contact model used in Section 3.4, which computes contact forces from interfacial stiffnesses and advances the equations of motion explicitly in time, PyBullet resolves contacts through an impulse-based constraint solver that corrects penetration through coupled velocity and position-level updates. Therefore, the VSR2

Contact physics parameters	Description	Unit
Restitution	Coefficient of restitution, measuring collision elasticity (bounciness), determining how much normal-direction kinetic energy is conserved (1 = perfectly elastic, 0 = perfectly inelastic).	Unitless
Lateral friction	Coulomb friction coefficient resisting tangential (sliding) motion at the contact patch.	Unitless
Spinning friction	Coefficient of friction opposing rotational motion around the contact normal.	Unitless
Contact damping	Damping factor for contact forces, reducing oscillations and stabilizing simulation by dissipating energy.	$N \cdot s \cdot m^{-1}$ (force per unit velocity)
Contact stiffness	Stiffness factor for contact forces, determining rigidity of contacts and affecting penetration deformation during contact.	$N \cdot m^{-1}$ (force per unit penetration distance)

Table 1 Primary body-level contact physics parameters for rock responses to ground motions. The corresponding effective rock–pedestal contact-pair values are obtained through PyBullet’s default contact-parameter combination rules, summarized in the Supplementary Material.

contact parameters should be interpreted as numerical controls within an impulse-based contact formulation rather than as direct counterparts to the force-based contact parameters used in DEM.

3.3 Parameter Optimization

We implemented an iterative parameter optimization procedure to calibrate the set of contact physics parameters in VSR2, with the objective of reproducing the overturning results observed in real shake-table experiments. In this procedure, one parameter was optimized at a time while all others were held constant. For each trial value, we compared the simulated overturning outcomes against the experimental results for the 582 recorded earthquake motions and quantified performance using classification-based metrics. The value that maximized the objective function was then selected and carried forward to the optimization of the next parameter.

Before introducing the detailed optimization process, we first defined the evaluation metrics using standard classification terms. Specifically, we categorized each simulation outcome as a true positive (TP), true negative (TN), false positive (FP), or false negative (FN), which were defined in Table 2. Based on these classifications, we calculated the following standard overturning performance metrics: accuracy = (TP + TN) / (TP + TN + FP + FN), precision = TP / (TP + FP), recall = TP / (TP + FN), and F1-score = 2 × (precision × recall) / (precision + recall). Accuracy reflects the overall agreement between simulation and experiment, but may be biased in the presence of an uneven dataset. High precision indicates a low rate of false positive alarms, while high recall reflects a low rate of false negatives, or missed failures. The F1-score provides an integrated evaluation that considers both precision and recall. In our optimization process, we defined the objective function as the average of accuracy, precision, and recall, assigning equal importance to each metric.

To implement the optimization, we conducted an iterative parameter search in which each of the five contact

Confusion matrix term	Definition
True positive	Overturn occurred in the real shake-table experiment and was correctly predicted by VSR2.
True negative	Stability (no overturn) observed in the real shake-table experiment and correctly predicted by VSR2.
False positive	Stability observed in the real experiment, but VSR2 incorrectly predicted an overturn.
False negative	Overturn occurred in the real experiment, but VSR2 incorrectly predicted stability.

Table 2 Confusion matrix term definition.

physics parameters was optimized sequentially. The full procedure is outlined in Algorithm 1. For each parameter (line 3), we fixed the value of all other parameters (line 4) and defined a search range and step size, resulting in candidate parameter sets (line 5). VSR2 then ran all 582 earthquake recordings in the all-earthquake-recordings motion mode for every candidate parameter set (line 6). During each run, VSR2 logged the full time series of rock motion, from which we determined the overturning outcome based on the final rock orientation. For example, overturning was declared when either the final roll or pitch deviated more than 55° from the original orientation. This threshold was selected within an experimentally confirmed range of 45°–60°. By comparing these outcomes with the real shake-table results, we computed the objective function for each candidate parameter set (line 7).

For each parameter, this process yielded a set of objective function values corresponding to different parameter values. The value that maximized the objective function was selected as the optimal setting and carried forward into subsequent optimization rounds (line 9).

Algorithm 1 Iterative Parameter Optimization

Input:
 Set of contact-physics parameters
 $\theta = \{\text{restitution, lateral friction, spinning friction, contact damping, contact stiffness}\}$
 convergence threshold ε

Output:
 Optimized parameter set θ^*

```

1: Initialize  $\theta$  with baseline values
2: repeat
3:   for each parameter  $\theta_i \in \theta$  do
4:     Fix all  $\theta_j \in \theta, j \neq i$ 
5:     for each candidate value  $v$  in the search range of  $\theta_i$  do
6:       Run VSR2 on all 582 earthquake recordings with  $\theta_i \leftarrow v$ 
7:       Compute objective  $J(v) = \frac{\text{Accuracy} + \text{Precision} + \text{Recall}}{3}$ 
8:     end for
9:      $v^* \leftarrow \arg \max_v J(v)$ 
10:    Update  $\theta_i \leftarrow v^*$ 
11:  end for
12:  Adjust the search ranges and step sizes for each  $\theta_i$  (typically narrow them)
13:   $\Delta \leftarrow \max_i J(\theta_i^{\text{new}}) - J(\theta_i^{\text{old}})$ 
14: until  $\Delta < \varepsilon$ 
15: return  $\theta^* \leftarrow \theta$ 

```

Each parameter search held the previously optimized parameters updated and fixed (line 10). After completing one full iteration through all five parameters, we manually adjusted the search ranges and step sizes (typically narrowing them) for the next iteration (line 12). Each successive iteration further refined the parameter values. The iterative process continued until the improvement in the objective function fell below a defined threshold, usually requiring four to five full iterations to converge (line 13). The complete results of the optimization process are provided in Supplementary Table S1.

3.4 Benchmarking

Using the optimized contact physics parameters, we evaluated the VSR2 performance in simulating overturning dynamics. Specifically, we ran all 582 earthquake recordings using the optimized VSR2 configuration and calculated the resulting performance metrics. These results were then compared with the DEM simulation outcomes and real shake-table experimental observations reported by Saifullah and Wittich (2023) to assess the dynamics fidelity of VSR2.

In addition to accuracy, we assessed the computational efficiency of VSR2. A subset of 15 representative ground motions was selected as input for this analysis. We tested two different PBR geometries: (1) a detailed surface mesh (specimen 1), characterized by complex geometry and contact points, and (2) a simplified rectangular prism ($1\text{ m} \times 1\text{ m} \times 3\text{ m}$), representing simplified geometry with well-defined contact points. We conducted overturning simulations using both VSR2 and DEM on similarly configured hardware platforms (Intel i7-11370H CPU with 32 GB RAM). This setup allowed the runtime comparison to evaluate the same prescribed overturning problems across various ground-motion durations, shaking intensities, and geometric complexities. For each ground motion, we

recorded the total runtime and compared the simulation efficiency across the two frameworks.

For the computational efficiency benchmark, the VSR2 simulations were conducted using the optimized contact parameters. We performed the DEM simulations using 3DEC (Itasca Consulting Group, Inc., 2016), which implements the 3D DEM originally formulated by Cundall and Strack (1979) and Hart et al. (1988). The rocks and pedestal were represented as rigid bodies, with deformation confined to the rock-pedestal interface. Contact detection in 3DEC is based on the common-plane logic, which represents interactions through primary contact types such as vertex-to-face and edge-to-edge contacts. Subcontacts are formed at vertices by triangulating the faces, and each subcontact is assigned an area for force calculation. In this study, interfacial behavior was modeled using the Coulomb-slip law, in which the elastic contact response is governed by the normal stiffness K_N (10 GPa/m) and shear stiffness K_S (1 GPa/m), with shear resistance controlled by the friction angle (40°). These values correspond to the DEM parameter set that produced the highest F1 score for Specimen 1 in Saifullah and Wittich (2023). Tensile strength, cohesion, and damping were set to zero following Saifullah and Wittich (2023). The equations of motion were then integrated explicitly using a finite-difference time-marching scheme.

3.5 Fragility Sensitivity

We investigated the effects of individual contact physics parameters on overturning behavior. For each parameter, we conducted 582 earthquake simulations across a range of values while holding all other parameters constant. Using the resulting binary overturning outcomes, we applied a two-variable logistic regression model, which estimates the probability of overturning as a logistic function of PGA and PGV/PGA, to derive the 95%

overturning probability boundary curves. We used the 95% overturning probability because this threshold has been adopted in previous PBR studies (Rood et al., 2022) and is consistent with traditional confidence thresholds used in statistical hypothesis testing (Baker et al., 2013; Hanks et al., 2013). This analysis enabled us to quantify how variations in each parameter shift the fragility boundary, offering insight into their respective roles in controlling overturning dynamics.

4 Results

4.1 Overturning Performance

The parameter optimization process yielded the following optimal body-level contact physics parameters: restitution = 0.35, lateral friction = 0.7, spinning friction = 0.5, contact damping = $1 \times 10^7 \text{ N} \cdot \text{s} \cdot \text{m}^{-1}$, and contact stiffness = $2.6 \times 10^9 \text{ N} \cdot \text{m}^{-1}$. Under PyBullet's default pairwise combination rules (as summarized in Supplementary Material S2), these body-level inputs correspond to effective restitution = 0.12, effective lateral friction = 0.49, effective spinning friction = 0.70, effective contact damping = $2.0 \times 10^7 \text{ N} \cdot \text{s} \cdot \text{m}^{-1}$, and effective contact stiffness = $1.3 \times 10^9 \text{ N} \cdot \text{m}^{-1}$. Table 3 presents a comparison of the overturning performance metrics between the VSR2 and the DEM simulation results. Our VSR2 method achieved slightly better performance across all evaluation metrics compared to the optimized DEM simulation. Fig. 4 illustrates the overturning results from VSR2, DEM, and real shake-table experiments. Visually, the performance of VSR2 and DEM appears closely matched. Near the overturning-stability boundary, VSR2 exhibits false positive predictions for small scaled PGA and false negative predictions for small PGV/PGA.

Simulation method	Accuracy	Precision	Recall	F1-score
VSR2	0.88	0.87	0.85	0.86
DEM	0.87	0.85	0.84	0.85

Table 3 Overturning simulation performance comparison between VSR2 and DEM (3DEC).

4.2 Computational Efficiency

We evaluated the computational efficiency of VSR2. Table 4 presents a comparison of wall-clock runtimes between VSR2 and DEM simulations. Overall, VSR2 demonstrated significantly higher computational efficiency than DEM. For the complex mesh PBR, which involves a large number of contact points, VSR2 was approximately 10^3 to 10^5 times faster. For the simple prism geometry, VSR2 achieved speedups of at least 10^2 times. These results are consistent with the known performance limitations of DEM, in which computational cost increases with the number of contacts because contact forces must be updated for each contact at every time step (Itasca Consulting Group, Inc., 2024). The substantial efficiency gain offered by VSR2 enables large-

scale parameter sweeps and Monte Carlo fragility analyses to be conducted on a single desktop machine, making it a practical tool for calibration and fragility model development.

4.3 Effects of Individual Physics Parameters

We investigated the effects of individual contact physics parameters on the overturning dynamics of PBRs. Fig. 5 presents the overturning results obtained by varying each parameter individually, while holding the remaining parameters fixed at their optimized values. In this way, we assessed the sensitivity of the optimized values. For each case, we applied logistic regression to the binary overturning outcomes and plotted the resulting 95% probability of overturning curves in Fig. 6, illustrating how the overturning boundary shifts in response to changes in each parameter. Among the parameters examined, restitution and spinning friction produced relatively minor changes in the overturning boundary within the tested ranges. Lateral friction produced a consistent shift in the boundary: larger lateral friction generally moved the 95% overturning boundary toward lower shaking intensities, indicating greater fragility. Contact damping and contact stiffness also influenced the overturning boundary, but changes in these parameters did not produce a consistent shift toward either greater stability or greater fragility across the tested ranges.

5 Discussion

5.1 Physical Interpretation and Fragility Sensitivity

From the parameter optimization, the effective restitution value (0.12) is lower than values commonly reported for rocking structures, which often fall in the range of 0.5–0.8 based on experimental studies (Čeh et al., 2018). However, the lower restitution may reflect both geometric effects of the natural PBR tested and possible micro-crushing at the rock-pedestal interface during impact, which reduces rebound. In laboratory settings, restitution is often measured using rounded granite blocks or simplified geometries, which produce higher values because they minimize surface asperities and crushing effects (Kalliontzis et al., 2016). In contrast, the natural PBR tested (Fig. 3) had irregular contact surfaces that likely promoted additional energy dissipation, making the lower restitution physically plausible in the context of our experiments. The relatively high contact damping value is consistent with this interpretation, as it effectively models additional energy dissipation observed during testing. Laboratory measurements of lateral friction coefficients span a wide range depending on surface smoothness (Byerlee, 1978), and our optimized effective value (0.49) falls within these ranges and represents a common contact surface, consistent with our experiment setting. Overall, while restitution is not identical to laboratory-measured values for simplified geometries, the optimized parameters are

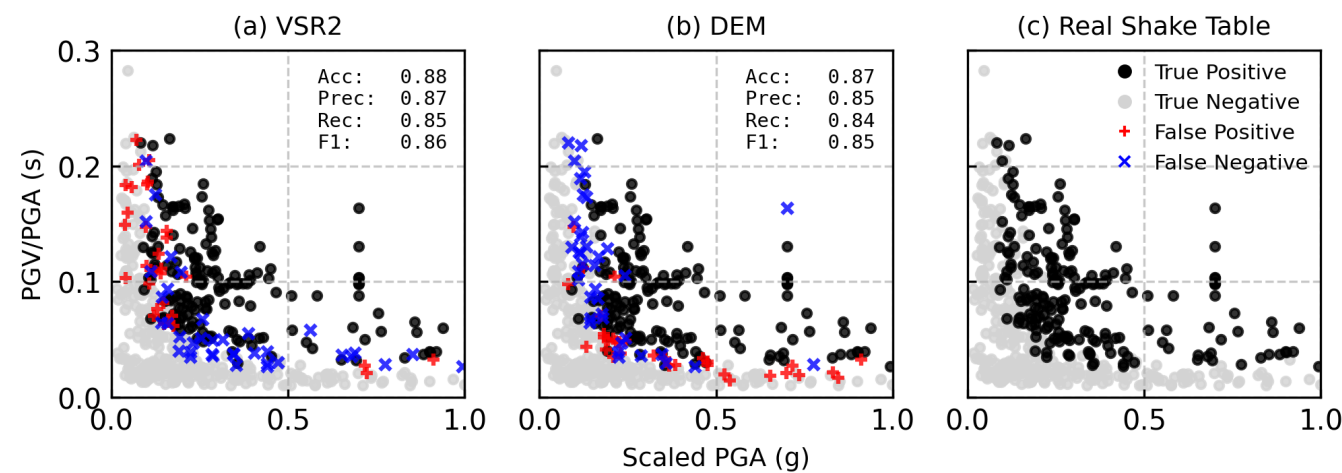


Figure 4 Overturning results from (a) VSR2, (b) discrete element method (DEM, using 3DEC), and (c) real shake-table experiments, plotted in scaled PGA–PGV/PGA space. See Table 2 for performance metrics. True positives indicate correctly predicted overturning, true negatives indicate correctly predicted stability, false negatives represent cases where overturning was incorrectly predicted as stability, and false positives represent cases where stability was incorrectly predicted as overturning. VSR2 demonstrates comparable predictive performance to DEM and strong agreement with real shake-table results.

Ground Motion	EQ Duration (s)	Mesh PBR			Prism (1 m × 1 m × 3 m)		
		VSR2 (s)	DEM (s)	DEM:VSR2	VSR2 (s)	DEM (s)	DEM:VSR2
Parkfield – 0.2g	21.1	1.6	2715	1697	1.2	600	500
Parkfield – 0.4g	21.1	1.6	2723	1702	1.2	600	500
Parkfield – 0.6g	21.1	1.5	9262	6175	1.2	381	318
Parkfield – 0.8g	21.1	1.5	3087	2058	1.2	380	317
Parkfield – 1.0g	21.1	1.6	4613	2883	1.2	469	391
Landers – 0.2g	58.6	4.4	4215	958	3.4	1156	340
Landers – 0.4g	58.6	4.6	45941	9987	3.5	969	277
Landers – 0.6g	58.6	4.3	46929	10914	3.6	1020	283
Landers – 0.8g	58.6	4.1	16955	4135	3.5	1023	292
Landers – 1.0g	58.6	4.5	9877	2195	3.6	1072	298
Chi-Chi – 0.2g	90.0	6.5	5226	804	6.2	1553	250
Chi-Chi – 0.4g	90.0	6.7	6217	928	5.5	1633	297
Chi-Chi – 0.6g	90.0	6.4	7170	1120	5.4	1642	304
Chi-Chi – 0.8g	90.0	6.3	4220	670	5.3	1599	302
Chi-Chi – 1.0g	90.0	6.6	24465	3707	5.5	1546	281

Table 4 Runtime comparison of VSR2 and DEM. Ground motions are selected from the NGA-West2 dataset (Ancheta et al., 2014) and then scaled.

physically plausible and align with the specific experimental conditions used in this study.

The sensitivity analysis shows that the inferred fragility boundary is not equally influenced by all contact-physics parameters, which provides insight into which aspects of rock-pedestal interaction are most important for overturning under single-axis excitation. In our simulations, increasing lateral friction shifted the 95% overturning boundary toward greater fragility, suggesting that frictional resistance at the rock-pedestal interface is an important control on overturning under the single-axis shaking conditions considered here. This behavior is consistent with prior rigid-block studies showing that friction governs the transition between sliding, rocking, and coupled sliding-rocking response, and that allowing sliding can modify the critical ground acceleration required for overturning

(Kounadis, 2018; Nodargi and Bisegna, 2025). Specifically, Kounadis (2018) showed that sliding can increase energy loss and may significantly increase the minimum ground-acceleration amplitude required to overturn multi-rigid-block assemblies. Similarly, Nodargi and Bisegna (2025) found that blocks with sufficiently low normalized friction can exhibit an isolation-like response, in which sliding prevents overturning but at the cost of larger permanent displacement.

Restitution produced comparably minor changes in the fragility boundary within the tested ranges. Restitution significantly affects the post-impact rocking response and the associated energy dissipation after rocking has already started, rather than the initiation of rocking itself (Kalliontzis et al., 2016; Čeh et al., 2018; Thomaidis et al., 2022). However, studies suggest that its effect on overall overturning behavior is more lim-

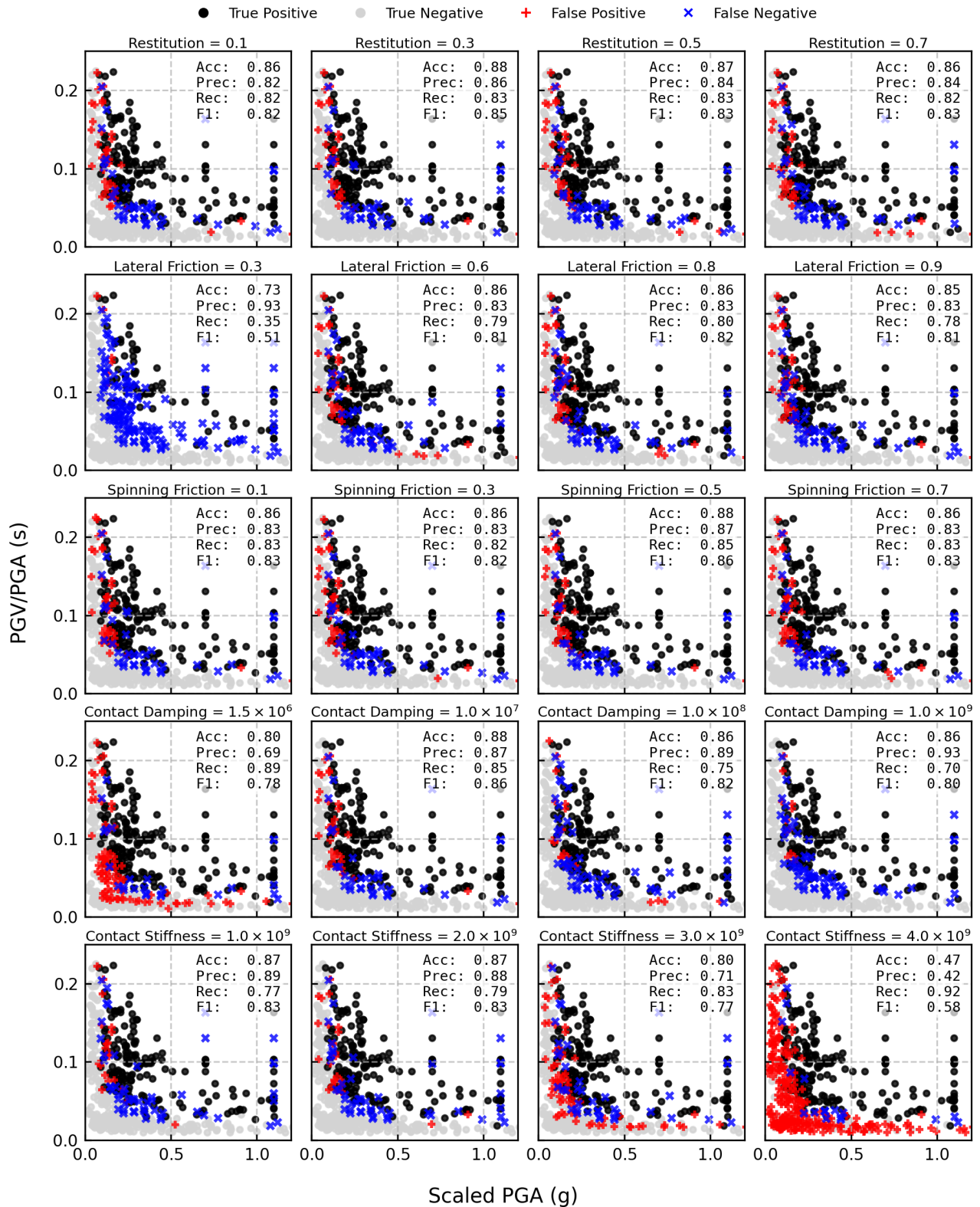


Figure 5 Overturning results from VSR2 under variation of individual contact physics parameters: restitution, lateral friction, spinning friction, contact damping, and contact stiffness. Each subplot in a row shows overturning outcomes plotted in PGV/PGA versus scaled PGA space for a specific parameter value, while keeping other parameters fixed at their optimized values. The variation illustrates the sensitivity of overturning prediction performance to each contact parameter.

ited than might be expected (Ishiyama, 1982; Yim et al., 1980). While our results are broadly consistent with this interpretation, PyBullet implements contact through an impulse-based algorithm in which restitution interacts with other contact parameters (contact damping and

contact stiffness), rather than through an explicitly isolated restitution model as in the aforementioned studies. Because the present study is based on a single PBR specimen, further experiments on a broader range of PBR geometries and contact conditions are needed to

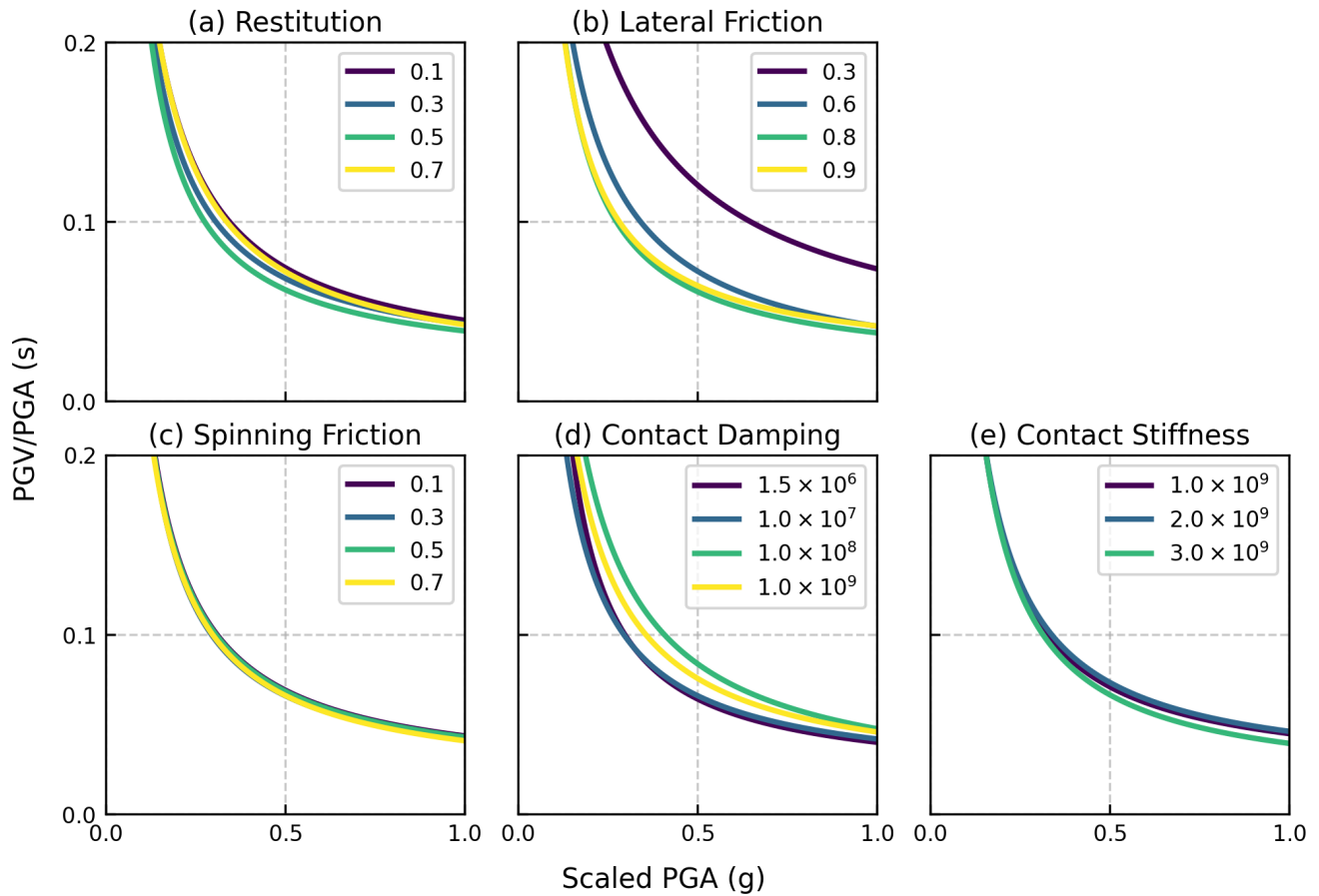


Figure 6 Logistic regression results showing the 95% overturning probability curves in scaled PGA–PGV/PGA space for different values of individual contact physics parameters. Each subplot varies one parameter group: (a) restitution, (b) lateral friction, (c) spinning friction, (d) contact damping ($N \cdot s \cdot m^{-1}$), and (e) contact stiffness ($N \cdot m^{-1}$), while holding all other parameters fixed at their optimized values. The curves represent the fragility boundaries from logistic models fitted to VSR2 simulation outcomes.

better assess the role of restitution in overturning. Spinning friction exhibited even weaker sensitivity in our experiments, indicating that torsional resistance about the contact normal played a secondary role in controlling overturning under single-axis shaking. This limited influence is likely attributable, at least in part, to the experiment setup. As shown in Fig. 3, the rock was placed in a relatively stable orientation that was less prone to twisting motion.

Contact damping and contact stiffness influenced the fragility boundary in a more complex manner. Increasing these parameters did not consistently move the boundary toward either higher or lower shaking intensities, indicating that their effects cannot be interpreted as simply stabilizing or destabilizing. This behavior is consistent with prior studies showing that rocking dynamics are highly sensitive to dissipation and contact modeling, and that nonlinear spring-damper representations of impact (contact) can introduce additional complexity into the response (Casapulla et al., 2017; Spanos et al., 2017; Galvez et al., 2022). Supplementary Material S2 further describes how contact damping and contact stiffness are incorporated into contact response in PyBullet. This formulation may contribute to the coupled and complex effects of these parameters observed

in our simulations.

5.2 Implications for PBR Fragility Modeling

The sensitivity results have important implications for how PBR fragility models should be calibrated and interpreted. Because the fragility boundary does not respond equally to all contact-physics parameters, uncertainty should not be treated uniformly across the parameter set. Parameters such as lateral friction, which strongly shift the fragility boundary, require closer calibration and more careful uncertainty characterization, whereas weakly sensitive parameters may be less identifiable from this class of experiments alone.

From an operational perspective, integrating VSR2-based fragility estimates into PSHA offers a promising pathway for regional hazard analysis. The calibrated contact parameters established in this study provide a baseline for refining parameter values for PBRs at new sites without requiring full recalibration in every case. More importantly, the computational efficiency of VSR2 makes it feasible to propagate uncertainty in influential contact parameters into uncertainty in the resulting fragility model. This capability is essential for using PBRs as quantitative ground-motion constraints, because hazard interpretation depends not only on the

nominal fragility boundary, but also on the uncertainty associated with that boundary. Efficient uncertainty-aware fragility modeling for larger ensembles of PBRs is therefore a necessary step toward integrating PBR constraints more systematically into probabilistic seismic hazard assessment.

5.3 Methodological Advancement

The development of VSR2 introduces a significant advancement in the simulation of overturning dynamics for PBRs. We develop VSR2 directly with PyBullet and ROS2, without relying on Gazebo, enabling higher numerical precision and more reliable tuning of contact physics parameters for accurate and reproducible simulations (building on the VSR introduced by [Chen et al., 2024a](#)). Its modular architecture, which leverages ROS2 namespaces and structured YAML configurations, enables flexible parameterization and reproducible experimentation. The support for multiple motion modes, including single cosine, grid cosine, single earthquake displacement recording, and all earthquake displacement recordings, allows the system to simulate a wide range of seismic loading conditions with high flexibility. VSR2 wall clock computation time for individual realizations is significantly faster than equivalent DEM calculations. Additionally, VSR2's parallel simulation platform dramatically improves computational throughput, making large-scale parameter sweeps and Monte Carlo fragility analyses feasible even on modest hardware. The Supplementary Material S4 includes details of the parallel simulation workflow and performance benchmarks. These technical innovations offer a scalable and extensible platform for virtual shake testing.

Contact overlap and timestep selection have different interpretations in VSR2 than in DEM, because the two methods use different contact formulations. In DEM, contact forces are computed from the overlap between bodies, then advanced with explicit time integration. The timestep must therefore be small enough to resolve the rapid contact response; otherwise, the explicit integration can produce unstable forces, velocities, or overlaps. In PyBullet, contacts are handled as constraints. When two bodies are detected in contact, PyBullet estimates contact points, contact normals, and penetration depth. The penetration depth is treated as a positional constraint error and enters the normal-contact impulse through a correction term scaled by the error reduction parameter, while contact stiffness and damping are mapped into the constraint force mixing and error reduction terms (Supplementary Material S2). Thus, contact overlap in PyBullet is used to correct constraint violation during the impulse-based contact solve, rather than being interpreted directly as physical deformation that generates a DEM-style contact force. DEM overlap criteria, such as limiting overlap to a fixed fraction of particle radius ([Hanley et al., 2015](#)), and the DEM critical-timestep criterion are not directly transferable to VSR2.

In VSR2, the timestep primarily controls the temporal resolution of the prescribed ground motion and impact events, whereas the number of solver iterations

affects convergence of the iterative constraint-solution process. Increasing the solver iteration count allows PyBullet to further reduce velocity and positional constraint errors within each timestep. In this study, VSR2 used a fixed timestep of 0.001 s and 150 solver iterations per timestep. The reproduced DEM simulations used a timestep calculated following the 3DEC procedure for explicit integration, in which the timestep is selected from the critical stability condition associated with the model's mass and stiffness.

5.4 Limitations and Future Work

A limitation of this study is that calibration was performed using a single natural PBR specimen. Although the system was evaluated with a comprehensive suite of 582 real earthquake ground motions, the use of only one specimen constrains the scope of dynamic behaviors represented in the calibration. Rock geometry and pedestal contact conditions vary in nature, and a single specimen may over-represent certain rock response behaviors to ground motion while under-representing others. For example, our experiments indicate that lateral friction increases resistance to sliding and thus increases the likelihood of overturning. This relationship, however, may strongly depend on the geometry of the rock, which requires more comprehensive evaluation. Future work should expand calibration and evaluation to multiple PBRs with varying geometries and pedestal conditions, enabling more robust characterization of contact parameter effects and improving the reliability of fragility assessments across rock populations. Additionally, although using the same 582-motion dataset for both calibration and benchmarking allowed direct comparison with the previous study, future work should adopt separate calibration and validation datasets, or cross-validation, to enable a more rigorous evaluation of out-of-sample predictive reliability. Future work should further examine how physics-engine simulation settings, such as timestep and solver iterations, affect PBR fragility modeling through timestep-sensitivity, solver-convergence, and frequency-domain analyses.

Although this study focused on rigid-body dynamics, future extensions should consider more complex rheological behaviors such as inelastic deformation and micro-crushing (e.g., continuous grain crushing, [Zhu and Zhao, 2019](#)). By implementing a state-dependent contact model within PyBullet, contact parameters are updated dynamically after each impact to reflect permanent indentation and energy loss. Reductions in stiffness and restitution, combined with increased damping, can emulate local crushing or compaction without altering body geometry. Alternatively, a thin compliant layer can be introduced beneath the rock to reproduce limited deformation at the interface.

While the current study focused on binary overturning outcomes, future work should also explore a broader range of dynamic rock behaviors. Future studies should extend the analysis beyond the single-axis excitation considered here to multi-axis ground motions, for which coupled translational and rotational

responses may change the interaction among rocking, sliding, twisting, and jumping. These behaviors may be more sensitive to certain ground-motion frequencies and could reveal more nuanced effects of contact physics parameters. Expanding VSR2 analysis in this direction will enhance its utility for both engineering design and earthquake science. For PSHA applications, future work should also quantify uncertainty in individual contact parameters and propagate that uncertainty into the hazard curve constraints.

6 Conclusion

This study evaluates a physics-engine approach for modeling the overturning behavior of PBRs under earthquake shaking and shows that it can reproduce experimental overturning outcomes with reliability comparable to DEM while greatly reducing computational cost. The results demonstrate that not all contact-physics parameters influence PBR fragility equally. Lateral friction exerted the strongest control on the inferred fragility boundary, whereas restitution and spinning friction had comparatively minor effects within the tested ranges, and contact damping and stiffness showed more complex influences. These findings indicate that single-axis overturning data constrain some contact parameters more strongly than others and that uncertainty in PBR fragility models should therefore not be treated uniformly across the parameter set. More broadly, these results support uncertainty-aware PBR fragility modeling. Because PBR-based ground-motion constraints depend not only on the nominal overturning boundary but also on the uncertainty associated with that boundary, sensitivity-informed calibration is important for seismic hazard applications. This study therefore provides a practical step toward using PBRs more systematically as quantitative constraints in probabilistic seismic hazard assessment.

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Data and code availability

The Virtual Shake Robot 2 Software for this study is available on Zenodo: <https://doi.org/10.5281/zenodo.19506439> (April 11, 2026).

Competing interests

The authors declare no competing interests.

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